A robust test of Merton's structural model for credit risk

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This paper presents a robust test of Merton's structural model for credit risk that does not depend on either estimated parameters for the firm's value or estimated default probabilities. We derive a test for the consistency of the changes in observed debt and equity prices (positive or negative changes) with the Merton model. For all firms selected and for all debt issues examined, the evidence strongly rejects Merton's structural model.

1. Introduction

Merton's (1974) structural model for credit risk model is based on the key insight that a firm's equity is analogous to a call option on the firm's assets. This simple call option analogy has spawned a vast academic literature studying its extensions and empirical implementation.[1] Furthermore, it has fostered an industry that uses this model to generate default likelihoods (see Kealhofer and Kurbat, 2001) employed to manage credit risk in banks and bond portfolios.

Merton's model invokes the arbitrage free pricing methodology in frictionless and competitive markets. The key characteristic of Merton's model is that the underlying state variable that determines a firm's default is the value of its assets. Default free interest rates are assumed to be constant. Given a specific balance sheet for the firm (a fixed and static structure), at the maturity date of its short-term debt (a discount bond), if the firm's value falls below the face value of the short-term debt, then default occurs. In the event of default, the payoffs to the firm's liabilities follow absolute priority (written into the debt's covenants). Under this structure, the firm's equity is analogous to a call option on the firm's assets.

As is well known, Merton's model has four empirical difficulties that make its implementation problematic:

Helpful comments from the finance workshop at Pennsylvania State University are gratefully acknowledged.

[1] References are provided below.

1. The term structure of default free interest rates is assumed to be constant.
2. The assets of the firm do not trade (in their entirety) and therefore their prices are not observable. This causes the underlying arbitrage free pricing argument to fail. From a practical perspective, it also makes it difficult (if not impossible) to estimate the parameters of the model for use in the valuation equations.
3. The liability structure of the firm is more complex than a single discount bond, and more importantly, the liability structures change over time.
4. In the event of default, the absolute priority structures are not adhered to by the bankruptcy courts, see Eberhart, Moore and Roenfeldt (1990), Weiss (1990). This misspecifies the resulting valuation formulas (and the inferred default probabilities).

Extensions and generalizations have been formulated to address many of these limitations. Stochastic interest rates were considered by Shimko, Tejima and van Deventer (1993), Nielsen, Saa-Requejo and Santa Clara (1993), and Longstaff and Schwartz (1995). Estimation issues by Duan (1994) and Ericsson and Reneby (2001, 2002). Models with more complex and dynamic liability structures include Jones, Mason and Rosenfeld (1984), Leland (1998), Tauren (1999), Colin-Dufresne and Goldstein (2001), and Hui, Lo and Tsang (2003). Last, Nielsen, Saa-Requejo and Santa Clara (1993), Longstaff and Schwartz (1995), Anderson and Sundaresan (1996) and Mella-Berral (1999) generate models that relax absolute priority.

Testing the Merton model is important from both an academic and institutional perspective. Indeed, the Basle Committee on Banking Supervision requires testing of credit models in its proposed New Basle Capital Accord: "A bank must demonstrate to its supervisor that the internal validation process enables it to assess the performance of internal rating and risk quantification systems consistently and meaningfully."[2]

Various tests of the structural approach to credit risk have been conducted. The evidence with respect to the consistency of the structural approach is mixed (see Jones, Mason and Rosenfeld, 1984; Delianedis and Geske, 1998; Jarrow and van Deventer, 1998, 1999; Eom, Helwege and Huang, 2000; Anderson and Sundaresan, 2000; Collin-Dufresne, Goldstein and Martin, 2001; Ericsson and Reneby, 2001; Kealhofer and Kurbat, 2001; and Huang and Huang, 2002). These studies, as common to most empirical investigations in finance, are really tests of a joint hypothesis: the structural model itself and the method for estimating the parameters. The method for estimating the parameters is especially important due to the fact that the firm's value is not observable (see Duan, 1994, and Ericsson and Reneby, 2001, 2002, in this regard).

[2] Section 302, p. 55, The New Basle Capital Accord, Basle Committee on Banking Supervision, May 31, 2001.

The purpose of this paper is to take "one-step back" from this escalating literature in order to provide a simple and robust test of the original Merton model. We design a test that does not depend on the parameter estimation procedure and is, therefore, not subject to "the curse of a joint hypothesis." We accomplish this by testing an implication of the model that is independent of the firm value's parameters, and that is based solely on observables. The well-known implication of the structural approach tested herein is that, ceteris paribus, as equity prices decrease, debt prices also decrease. In street language, a roughly equivalent statement is that as equity prices decrease, credit spreads rise.[3]

Unfortunately, due to the differential liquidity of equity versus bond markets, the use of market prices introduces micro-structure considerations into the analysis. Bond markets are usually considered less liquid. Illiquid bond markets introduce observational error or noise into available market prices. We address these market micro-structure considerations in two ways. First, we use only weekly and monthly observation intervals in our testing procedure in order to reduce the impact of daily price illiquidities. Second, we design a statistical methodology that explicitly incorporates micro-structure observational error.

Using available debt and equity prices from Bank One Corporation, Enron, Exxon, Merrill Lynch, and Whirlpool over the time period February 6, 1992, to March 12, 2001, we check for the consistency of the Merton model using two (non-parametric) tests. The first test is based on changing debt and equity prices, and the second test is based on changing credit spreads and equity prices. The structural approach is rejected in all cases for all bond issues for both tests using weekly observation intervals. For monthly observation intervals, it is rejected for almost all bond issues for both tests. In these exceptions, the inconsistencies were usually high, but the sample size too small to generate a rejection of the null hypothesis.

For example, for Bank One, the average percentage of weekly sign changes of debt versus equity prices inconsistent with the Merton model is 47%. Taking the overly conservative position that up to 20% of the price observations are in error, in all cases, the Merton model is strongly rejected. The probability of rejecting the Merton model when it is true (the type I error) is negligible (exceeding five standard errors in all cases). For the second test, Bank One's average percentage of weekly sign changes for credit spreads versus equity prices inconsistent with the Merton model is 49%. Assuming the same error structure, the Merton model is rejected again (exceeding two standard errors in all cases).

An outline for this paper is as follows. Section 2 presents the structural model and derives the testable implication. Section 3 derives the statistical model. Section 4 provides the empirical evidence, while Section 5 concludes the paper.

[3] We will subsequently show that this translation to credit spreads is not precisely true, although the first statement in terms of debt prices is exact.

2. Merton's model

Merton's model assumes frictionless, competitive, arbitrage-free, complete markets, and that the firm's balance sheet can be written as:

Assets	Liabilities
V_t	$D_t\,(T, F)$
	E_t

where V_t is the value of the firm's assets at time t assumed to follow a diffusion process, D_t is the value of the firm's debt at time t, and E_t is the value of the firm's equity at time t. The debt is assumed to be a zero-coupon bond with maturity T and face value F.

Let r_t denote the (default free) spot rate of interest at time t, assumed to be non-random.

Using the call option analogy to the firm's equity, Merton (1974) develops a pricing formula for debt $D_t = D_t(V_t, r_t\colon T, F)$ that depends on the firm's value (and the parameters of its stochastic process – the volatility), the characteristics of the debt (face value and maturity date), and the spot rate of interest (r_t).

To following testable implication follows directly from Merton's formula (see the appendix for the exact formulas).

PROPOSITION 1 (DEBT PRICES) *Given frictionless, competitive, arbitrage-free and complete markets, if*

1. V_t *is a diffusion process,*
2. $D_t = D_t(V_t, r_t)$ *and* $E_t = E_t(V_t, r_t)$,
3. $\partial D_t(V_t, r_t)/\partial V_t > 0$ *and* $\partial E_t(V_t, r_t)/\partial V_t > 0$,
4. r_t *is non-random, and*
5. *time t does not correspond to the maturity date of the debt, a coupon payment date, or a dividend payment date,*

then $\operatorname{sign}(\mathrm{d}D_t - r_t D_t \mathrm{d}t) = \operatorname{sign}(\mathrm{d}E_t - r_t E_t \mathrm{d}t)$.

The proof is contained in an appendix.

The testable implication of the model in Proposition 1 is that the sign of the change in equity prices, after an adjustment for the spot rate of interest, must equal the sign of the change in debt prices. This implication is independent of observing the firm's value or the parameters of its diffusion process.

It is important to note that the maintained hypotheses of frictionless and competitive markets underlying Proposition 1 excludes liquidity risk as a partial determinate of debt (and equity) prices. In the current credit risk literature, there is strong evidence supporting the contention that the credit spread includes a liquidity risk premium (for example, see Liu, Longstaff, Mandell, 2001; and Janosi, Jarrow, Yildirim, 2002). This omission could be one of the reasons for the rejection of Proposition 1 (and 2) below.

As written, we also see that this testable implication is consistent with a larger class of structural models than that contained in Merton (1974). This implication depends on only four hypotheses. The first hypothesis is that the firm's value follows a diffusion process. The second hypothesis is that both debt and equity can be written as (twice continuously differentiable) functions of the firm's value. The third hypothesis is that the value of both debt and equity increase as the firm's value increases. The fourth hypothesis is that the spot rate is non-random.[4] The fifth condition listed in the Proposition is that there are no cashflows to debt or equity on the date for which the sign is measured. This condition speaks to the empirical methodology and it not a hypothesis pertaining to the underlying model.

The extended class of Merton models allowed includes those where the asset value process is not lognormally distributed, but can follow an arbitrary diffusion process,[5] and those where the firm's liabilities can consist of a collection of coupon bonds (of equal seniority), but with different maturities and face values. Although the liability structure need not consist of a single discount debt issue, the liability structure must still be static and adhere to absolute priority. However as in the original Merton model, the third hypothesis is quite restrictive. It excludes those generalizations where equity values increase at the expense of debt.

It is more difficult to get a test based on credit spreads. Let us define the yield on the firm's debt as $y_t \equiv -(\log D_t)/(T-t)$, and the credit spread as $c_t \equiv y_t - r_t$.

PROPOSITION 2 (CREDIT SPREADS) *Let the same hypotheses hold as in Proposition 1. If* $\text{sign}(\text{d}E_t - r_t E_t \text{d}t) < 0$, *then* $\text{sign}(\text{d}c_t) > 0$. *But if* $\text{sign}(\text{d}E_t - r_t E_t \text{d}t) > 0$, *then* $\text{sign}(\text{d}c_t)$ *is ambiguous. It depends on size of the positive stock price increase.*

The proof is contained in the Appendix.

Unlike Proposition 1, this Proposition gives an implication of the structural model that is only one sided. As equity prices decrease, the credit spread increases. However, if equity prices increase, the sign of the change in the credit spread is ambiguous. This is due to the fact that the bond's yield is a convex function of the debt's price. In essence, if equity prices increase, but only by a small amount, the credit spread could still increase due to the passage of time and the non-linearity present in the computation of a bond's yield.

3. The statistical model

This section formulates a non-parametric statistical test for Propositions 1 and 2. If the price observations contained no noise (observational error), then the prob-

[4] Although the implication of Proposition 1 is consistent with random interest rates, a careful examination of the proof indicates that the Proposition does not generalize for this extension.

[5] This is true only to the extent that hypothesis (3) is satisfied by the diffusion process selected.

ability of rejecting the null hypothesis (the Merton model) when it is true, given even one inconsistent observation, is zero. Under this belief, there is no Type I error. This statement is the analogue of being able to reject the hypothesis of no arbitrage in an economy (for sure), if there is just one observation of arbitrage. Yet, we do not believe that the price observations are observed without error. When we see an inconsistent observation, it is either because there is noise in the observation or the null hypothesis is false.

Let us suppose that α is the percentage of inconsistent observations due to noise when testing either Proposition 1 or 2. Then, under the null hypothesis of the Merton model, the true percentage of inconsistencies[6] in a sample size of n, denoted $\bar{p}$, follows a binomial distribution[7] with mean α and standard deviation $\sqrt{\alpha(1-\alpha)/n}$.

Let $\hat{p}$ denote the percentage of inconsistencies actually observed in the data. The probability of rejecting the Merton model when it is true at the $\hat{p}$ level, under this error structure, is equal to:

$$\Pr\{\bar{p} \geq \hat{p}\} = \Pr\left\{\frac{\bar{p}-\alpha}{\sqrt{\alpha(1-\alpha)/n}} \geq \frac{\hat{p}-\alpha}{\sqrt{\alpha(1-\alpha)/n}}\right\}$$

$$\approx 1 - N\left(\frac{\hat{p}-\alpha}{\sqrt{\alpha(1-\alpha)/n}}\right) = N\left(-\frac{\hat{p}-\alpha}{\sqrt{\alpha(1-\alpha)/n}}\right)$$

where $\Pr\{\cdot\}$ is a binomial distribution with mean α and standard deviation $\sqrt{\alpha(1-\alpha)/n}$, and $N(\cdot)$ is the cumulative standard normal distribution function.

We see here that

$$\frac{\hat{p}-\alpha}{\sqrt{\alpha(1-\alpha)/n}}$$

is the number of standard deviations to the left of the mean in a standard normal distribution that the observed inconsistencies represent.

Let us assume, as an over-estimate (to be conservative), that noise accounts for 20% of the inconsistent price observations. Then, the number of standard deviations to the left of the mean that the actual percentage inconsistencies represent is $\sqrt{n}(\hat{p}-0.2)/(0.4)$. For a one-sided test of the null hypothesis the z-scores are: 1.282 for 10%, 1.45 for 7.5%, and 1.645 for 5%.

Assuming that 20% of the inconsistent price observations represent noise is a conservative assumption when using weekly or monthly observation intervals. The true error within the data is probably closer to zero, however, if the Merton model can be rejected using this conservative error structure, it will provide a convincing rejection.

[6] This is the sample mean of observing either a 1 (inconsistent) or a 0 (consistent) observation.

[7] By the Central Limit Theorem, as $n \to \infty$, we can approximate the binomial with a normal distribution.

4. Empirical evidence

The data is debt and equity prices obtained from Bridge Information Systems and Data Stream over the time period February 6, 1992, to March 12, 2001. All equity prices are obtained from Yahoo and they correspond to daily closing prices. Three different companies' various debt issues (Bank One Corporation, Enron, and Whirlpool) are from Data Stream, and two companies' debt issues (Exxon and Merrill Lynch) are from Bridge Information Systems. Data Stream is daily data and Bridge Information Systems is weekly data. These firms were chosen because their names are well-known and they span various industries. Daily US Treasury yields were obtained from the H15 statistical release from the Federal Reserve Board of Governors.

A concern with using daily observation intervals for debt prices is that daily price changes may contain significant market microstructure related noise (greater than $\alpha = 0.2$).[8] This would be a valid concern, for example, if the debt issues were illiquid.[9] Then, equity prices may change (due to their trading in liquid markets), but the debt prices or credit spreads (due to their trading in illiquid markets) may not change. This would generate an inconsistent observation in the testing of Propositions 1 or 2 due to the noise in the data. To reduce this concern, we report our results for only weekly and monthly observation intervals.[10]

To test Proposition 1, we looked at the weekly and monthly changes in equity prices and debt prices, after adjusting for the spot rate of interest. Debt prices for the coupon bonds are directly observable, and the addition of accrued interest is exactly specified by bond market rules. The spot rate of interest corresponds to the 3 month Treasury bill yield.

To test Proposition 2, we again looked at weekly and monthly changes in equity prices and compared them to changes in the credit spread. The credit spread was computed by determining that spread to Treasuries, $c(T)$ – a function of maturity, that best fits the observed corporate coupon bond prices. Obtaining the credit spread curve is a two-step procedure. In step one, the Treasury yield curve is obtained by fitting a third degree spline of maximum smoothness (see Adams and van Deventer, 1994) to the available Treasury yields. In step two, the credit spread curve is obtained by fitting the best spline of third degree, that when added to the Treasury yield curve, minimizes the sum of squared errors in fitting the corporate debt prices. Then, each bond's credit spread for testing Proposition 2 is that credit spread associated with its maturity. Since the test of

[8] Another underlying issue is that many of the bonds considered contain embedded options. Embedded options are another reason why the simple Merton model may not provide a good characterization of debt prices.

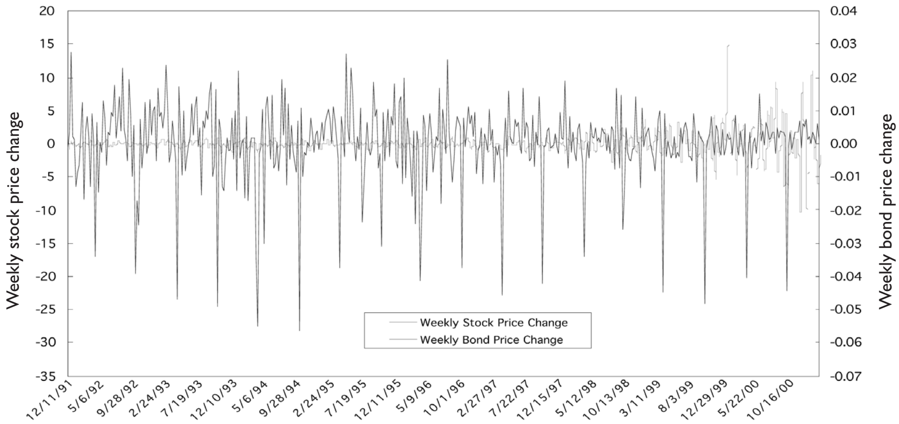
[9] This liquidity issue is distinct from that which was discussed following the statement of Proposition 1. The discussion following Proposition 1 related to the existence of a liquidity risk premium. The discussion here relates to the existence of liquidity-induced noise in the observation of both debt and equity prices.

[10] Daily results reject Merton's model with even higher levels of confidence then either the weekly or monthly observation interval cases.

Proposition 2 involves estimating a non-observable – the credit spread – this provides a less robust test of the null hypothesis. It is included because the industry typically "thinks" in terms of credit spreads.

Before discussing the evidence, to illustrate the testing procedure, we provide two figures. Figure 1 gives a representative time series graph of weekly stock price versus bond price changes for the 8.25% Enron bonds due September 15, 2012. Under Proposition 1, when equity prices decline we would expect to see

FIGURE 1 Enron stock price change versus bond price change on 9.13% due April 1, 2003, December 1991 to April 2001.



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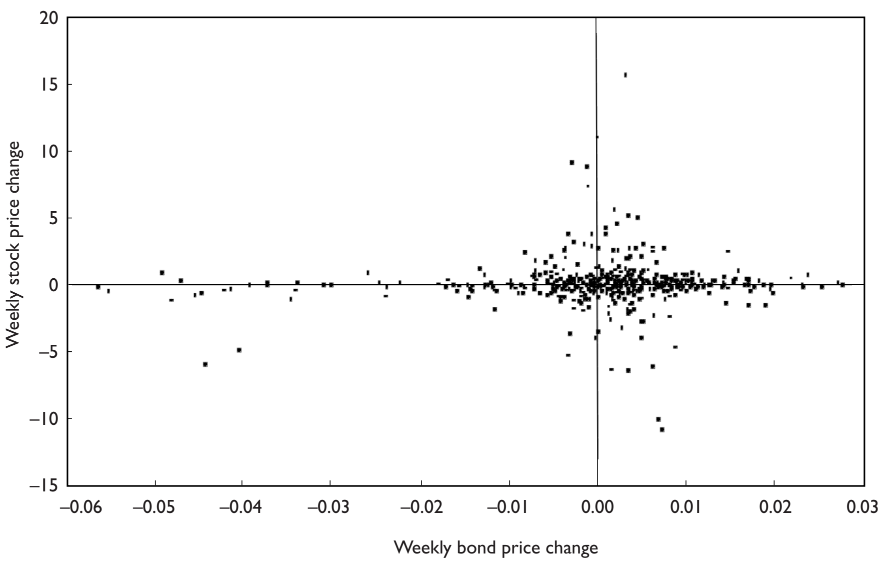

TABLE 1 Bank One Corporation (Weekly)

Issues	**10% 059438AB7 due 8/15/2010**	**7.25% 059438AD3 due 8/1/2002**	**8.74% 059438AE1 due 9/15/2003**	**7% 059438AF8 due 7/15/2005**	**7.75% 59438AG6 due 7/15/2025**	**7.63% 059438AH4 due 10/15/2026**	**7.6% 059438AJ0 due 5/1/2007**
Proposition 1							
Inconsistency	37.57%	38.70%	41.62%	40.00%	39.77%	48.21%	45.65%
Std deviations	8.26	8.40	10.23	6.71	6.56	7.46	6.15
Observations	354	323	358	180	176	112	92
Proposition 2							
Inconsistency	46.63%	50.00%	51.68%	49.38%	48.15%	56.25%	46.51%
Std deviations	8.50	9.37	9.67	6.61	6.33	6.28	4.49
Observations	163	156	149	81	81	48	46
Date information							
Starting date	1/2/92	8/3/92	12/4/91	7/21/95	7/21/95	10/25/96	4/25/97
Ending date	3/18/99	3/18/99	3/18/99	3/18/99	3/18/99	3/18/99	3/18/99

Issues	**8% 059438AK7 due 4/29/2027**	**9.88% 059438AL5 due 3/1/2019**	**6.6% 05943FAC9 due 3/23/2001**	**7% 05943FAD7 due 3/25/2002**	**6.59% 05943FAJ4 due 9/21/2001**	**6.25% 05943FAK1 due 10/1/2001**	**6.38% 5943FAM7 due 10/1/2002**
Proposition 1							
Inconsistency	49.45%	61.70%	50.00%	47.37%	46.38%	50.70%	57.14%
Std deviations	7.02	7.15	7.27	6.67	5.48	6.47	7.77
Observations	91	47	94	95	69	71	70
Proposition 2							
Inconsistency	50.00%	47.83%	48.89%	54.55%	45.16%	36.36%	48.48%
Std deviations	4.86	3.34	4.85	5.73	3.50	2.35	4.09
Observations	42	23	45	44	31	33	33
Date information							
Starting date	4/28/97	3/25/98	3/25/97	3/25/97	9/26/97	9/23/97	9/30/97
Ending date	3/18/99	3/18/99	3/18/99	3/18/99	3/15/99	3/18/99	3/18/99

Average of inconsistency for Proposition 1: 46.73%; Average of inconsistency for Proposition 2: 48.56%

TABLE 2 Enron (Weekly)

Issues	**9.88% 293561AF3 due on 6/15/2003**	**9.65% 293561AH9 due on 5/15/2001**	**9.5% 293561AN6 due on 6/15/2001**	**9.13% 293561AQ9 due on 4/1/2003**	**7.63% 293561AR7 due on 9/10/2004**	**8.25% 293561AS5 due on 9/15/2012**	**6.75% 293561AT3 due on 7/1/2005**	**7% 293561AU0 due on 8/15/23**
Proposition 1								
Inconsistency	45.43%	47.03%	44.77%	46.27%	45.24%	43.51%	47.24%	39.20%
Standard deviations	13.64	14.41	13.12	14.02	12.93	11.99	13.29	6.37
Observations	460	455	449	456	420	416	381	176
Proposition 2								
Inconsistency	48.15%	52.34%	51.39%	50.93%	45.77%	48.99%	49.16%	55.56%
Standard deviations	10.34	11.83	11.53	11.36	9.13	10.20	9.75	8.43
Observations	216	214	216	216	201	198	179	90
Date information								
Starting date	12/4/91	1/2/92	2/6/92	12/4/91	8/31/92	9/22/92	6/18/93	8/19/93
Ending date	3/12/01	3/12/01	3/12/01	3/12/01	3/12/01	3/12/01	3/12/01	4/14/97

Average of inconsistency for Proposition 1: 44.841%
Average of inconsistency for Proposition 2: 50.29%

TABLE 3 Exxon (Weekly)

Issues	6% 190157 due on 7/1/2005	6.15% 383210 due on 3/11/2003	6.13% 384907 due on 9/8/2008
Proposition1			
Inconsistency	43.91%	44.35%	44.35%
No. of standard deviations	9.07	9.23	9.23
No. of observations	230	230	230
Proposition2			
Inconsistency	48.54%	55.34%	50.49%
No. of standard deviations	7.24	8.97	7.74
No. of observations	103	103	103
Date information			
Starting date	4/18/95	4/18/95	4/18/95
Ending date	9/28/99	9/28/99	9/28/99

Average of inconsistency for Proposition 1: 44.20%
Average of inconsistency for Proposition 2: 51.46%

bond prices rise, and conversely. A careful examination of this graph reveals numerous instances where the equity price change and the bond price change are on opposite sides of the x-axis (a zero change), contradicting Proposition 1. Figure 2 further illustrates this inconsistency by graphing the ordered pairs (bond price changes, stock price changes) for this same Enron bond issue over the same time period. According to Proposition 1, all of the ordered pairs plotted should appear in only the second and fourth quadrant. This is clearly not the case.

Tables 1–5 contain the tests of both Propositions 1 and 2 for the five firms using weekly observation intervals. Given in the tables is the percentage of inconsistencies observed for each of the various bond issues. The starting date, ending date, and the number of observations used for each Proposition are listed. Also provided is the number of standard deviations that the observed percentage inconsistency exceeds the assumed percentage error of 20%. For example, consider Enron's results as given in Table 2. The first column is the 9.88% bond due on June 15, 2003. For the test of Proposition 1, 45.43% of the observations are inconsistent with Merton's model. The null hypothesis can be rejected at the 13.64 standard error level. For the test of Proposition 2, 48.15% of the observations are inconsistent with Merton's model at the 10.34 standard error level.

Glancing through the tables one sees that in all cases – for all firms, all debt issues, all sub-intervals – the sign changes of debt and equity prices are inconsistent with the Merton model and this inconsistency is not caused by chance. This evidence provides a strong rejection of the Merton model.

TABLE 4 Merrill Lynch (Weekly)

Issues	**7.25% 816021 due on 5/2/2002**	**7% 817342 due on 6/25/2004**	**6.5% 838204 due on 8/21/2001**	**6.75% 838875 due on 9/24/2004**	**6.25% 851579 due on 11/15/2000**
Proposition 1					
Inconsistency	44.00%	48.72%	53.27%	49.51%	41.87%
Std deviations	6.71	7.77	8.60	7.49	7.79
Observations	125	117	107	103	203
Proposition 2					
Inconsistency	45.76%	52.54%	52.83%	46.16%	53.12%
Std deviations	4.95	6.25	5.98	4.72	8.11
Observations	59	59	53	52	96
Date Information					
Starting date	4/29/97	6/24/97	9/2/97	9/30/97	10/31/95
Ending date	9/28/99	9/28/99	9/28/99	9/28/99	9/28/99

Average of inconsistency for Proposition 1: 47.47%
Average of inconsistency for Proposition 2: 50.08%

TABLE 5 Whirlpool (Weekly)

Issues	**9.02% 96332HAS8 due on 3/15/2001**	**9.1% 96332HAR0 due on 3/15/2004**	**9.14% 96332HAQ2 due on 3/15/2006**	**9.0% 963320AG1 due on 3/1/2003**	**9.1% 963320AE6 due on 2/1/2008**
Proposition 1					
Inconsistency	47.50%	46.25%	45.00%	43.51%	37.77%
Std deviations	6.15	5.87	5.59	7.29	6.09
Observations	80	80	80	154	188
Proposition 2					
Inconsistency	67.57%	54.06%	48.65%	50.00%	47.37%
Std deviations	7.23	5.18	4.36	6.36	6.67
Observations	37	37	37	72	95
Date information					
Starting date	5/17/93	5/17/93	5/17/93	12/5/91	3/4/91
Ending date	12/29/94	12/29/94	12/29/94	12/29/94	12/29/94

Average of inconsistency for Proposition 1: 44.01%
Average of inconsistency for Proposition 2: 53.53%

TABLE 6 Bank One Corporation (Monthly)

Issues	**10% 059438AB7 due 8/15/2010**	**7.25% 059438AD3 due 8/1/2002**	**8.74% 059438AE1 due 9/15/2003**	**7% 059438AF8 due 7/15/2005**	**7.75% 59438AG6 due 7/15/2025**	**7.63% 059438AH4 due 10/15/2026**	**7.6% 059438AJ0 due 5/1/2007**
Proposition 1							
Inconsistency	32.93%	45.45%	35.71%	41.46%	31.71%	11.11%	45.45%
Std deviations	2.93	5.58	3.60	3.44	1.87	-1.15	2.98
Observations	82	77	84	41	41	27	22
Proposition 2							
Inconsistency	44.74%	52.78%	50.00%	43.75%	47.67%	36.36%	14.29%
Std deviations	3.81	4.92	4.37	2.38	2.68	1.36	-0.38
Observations	38	36	34	16	15	11	7
Date information							
Starting date	1/2/92	8/3/92	12/4/91	7/21/95	7/21/95	10/25/96	4/25/97
Ending date	3/18/99	3/18/99	3/18/99	3/18/99	3/18/99	3/18/99	3/18/99

Issues	**8% 059438AK7 due 4/29/2027**	**9.88% 059438AL5 due 3/1/2019**	**6.6% 05943FAC9 due 3/23/2001**	**7% 05943FAD7 due 3/25/2002**	**6.59% 05943FAJ4 due 9/21/2001**	**6.25% 05943FAK1 due 10/1/2001**	**6.38% 5943FAM7 due 10/1/2002**
Proposition 1							
Inconsistency	18.18%	45.45%	45.45%	47.83%	41.18%	64.71%	70.89%
Std deviations	-0.21	2.11	2.98	3.34	2.18	4.61	5.25
Observations	22	11	22	23	17	17	17
Proposition 2							
Inconsistency	14.29%	66.67%	71.43%	85.71%	57.14%	37.50%	28.57%
Std deviations	-0.38	2.86	3.40	4.35	2.46	1.24	0.57
Observations	7	6	7	7	7	8	7
Date information							
Starting date	4/28/97	3/25/98	3/25/97	3/25/97	9/26/97	9/23/97	9/30/97
Ending date	3/18/99	3/18/99	3/18/99	3/18/99	3/15/99	3/18/99	3/18/99

Average of inconsistency for Proposition 1: 41.25%; Average of inconsistency for Proposition 2: 46.49%

TABLE 7 Enron (Monthly)

Issues	**9.88% 293561AF3 due on 6/15/2003**	**9.65% 293561AH9 due on 5/15/2001**	**9.5% 293561AN6 due on 6/15/2001**	**9.13% 293561AQ9 due on 4/1/2003**	**7.63% 293561AR7 due on 9/10/2004**	**8.25% 293561AS5 due on 9/15/2012**	**6.75% 293561AT3 due on 7/1/2005**	**7% 293561AU0 due on 8/15/23**
Proposition 1								
Inconsistency	46.87%	49.54%	46.23%	41.12%	47.96%	48.45%	40.90%	35.00%
Standard deviations	6.98	7.64	6.75	5.46	6.92	7.01	4.90	2.37
Observations	108	107	106	107	98	97	88	40
Proposition 2								
Inconsistency	49.02%	45.10%	51.02%	49.02%	57.89%	61.90%	40.00%	35.00%
Standard deviations	5.18	4.48	5.43	5.18	5.84	6.79	3.16	1.68
Observations	51	51	49	51	38	42	40	20
Date information								
Starting date	12/4/91	1/2/92	2/6/92	12/4/91	8/31/92	9/22/92	6/18/93	8/19/93
Ending date	3/12/01	3/12/01	3/12/01	3/12/01	3/12/01	3/12/01	3/12/01	4/14/97

Average of inconsistency for Proposition 1: 44.51%
Average of inconsistency for Proposition 2: 48.62%

TABLE 8 Exxon (Monthly)

Issues	6% 190157 due on 7/1/2005	6.15% 383210 due on 3/11/2003	6.13% 384907 due on 9/8/2008
Proposition1			
Inconsistency	46.55%	50.00%	39.66%
No. of standard deviations	5.05	5.71	3.74
No. of observations	58	58	58
Proposition2			
Inconsistency	70.00%	55.00%	45.00%
No. of standard deviations	5.59	3.91	2.80
No. of observations	20	20	20
Date information			
Starting date	4/18/95	4/18/95	4/18/95
Ending date	9/28/99	9/28/99	9/28/99

Average of inconsistency for Proposition 1: 45.40%
Average of inconsistency for Proposition 2: 56.67%

TABLE 9 Merrill Lynch (Monthly)

Issues	7.25% 816021 due on 5/2/2002	7% 817342 due on 6/25/2004	6.5% 838204 due on 8/21/2001	6.75% 838875 due on 9/24/2004	6.25% 851579 due on 11/15/2000
Proposition 1					
Inconsistency	53.12%	56.67%	55.56%	50.00%	49.02%
Std deviations	4.68	5.02	4.62	3.82	5.18
Observations	32	30	27	26	51
Proposition 2					
Inconsistency	58.33%	64.29%	58.33%	23.08%	33.33%
Std deviations	3.32	4.14	3.32	0.28	1.53
Observations	12	14	12	13	21
Date information					
Starting date	4/29/97	6/24/97	9/2/97	9/30/97	10/31/95
Ending date	9/28/99	9/28/99	9/28/99	9/28/99	9/28/99

Average of inconsistency for Proposition 1: 52.87%
Average of inconsistency for Proposition 2: 47.47%

TABLE 10 Whirlpool (Monthly)

Issues	9.02% 96332HAS8 due on 3/15/2001	9.1% 96332HAR0 due on 3/15/2004	9.14% 96332HAQ2 due on 3/15/2006	9.0% 963320AG1 due on 3/1/2003	9.1% 963320AE6 due on 2/1/2008
Proposition 1					
Inconsistency	31.58%	31.58%	31.58%	28.57%	43.18%
Std deviations	1.26	1.26	1.26	1.27	3.84
Observations	19	19	19	35	44
Proposition 2					
Inconsistency	66.67%	83.33%	33.33%	35.30%	52.63%
Std deviations	2.86	3.88	0.82	1.528	3.56
Observations	6	6	6	17	19
Date information					
Starting date	5/17/93	5/17/93	5/17/93	12/5/91	3/4/91
Ending date	12/29/94	12/29/94	12/29/94	12/29/94	12/29/94

Average of inconsistency for Proposition 1: 33.30%
Average of inconsistency for Proposition 2: 54.25%

Next, Tables 6–12 contain the tests using a monthly observation interval. A comparison with their weekly counterparts in Tables 1–5 shows that the average percentage inconsistencies are of similar magnitudes. This similarity of the weekly and monthly results is consistent with the observational error due to market microstructure considerations being minimal (and $\alpha = 0.2$ being excessive). Second, examining the tables for the test of Proposition 1, one sees that in all cases – for all firms, all debt issues, all sub-intervals – the sign changes of debt prices and equity prices are inconsistent with the Merton model at the 90% confidence level except for 2 out of 14 bond issues for Bank One and 4 out of 5 bond issues for Whirlpool. For Whirlpool, however, all of these 4 bond issues are at the 89% confidence level. Due to the decreased sample size, the rejection level is less than for the weekly observation intervals.

For the test of Proposition 2, again – for all firms, all debt issues, all sub-intervals – except for 4 out of 14 bond issues for Bank One, 1 out of 5 bond issues for Merrill Lynch, and 1 out of 5 issues from Whirlpool, the sign of the changes of credit spreads and equity prices are inconsistent with the Merton model at the 90% confidence level. This is strong evidence inconsistent with Merton model.

5. Conclusion

This paper derives and implements a robust test of Merton's structural model for valuing credit risk that does not involve estimating the firm's value, the parameters of the firm value's stochastic process, or implied default probabilities.

The testing methodology only requires the observation of equity prices, debt prices, and the spot rate of interest. We tested the Merton model on five different companies' debt issues in various sub-intervals using both weekly and monthly observation intervals over the time period February 6, 1992, to March 12, 2001.

Assuming that noise represents 20% of the inconsistencies observed, an overly conservative assumption, using weekly observation intervals, tests of Proposition 1 and 2 reject the Merton model for all the firms for all the debt issues for all time periods. This is a strong rejection. Using monthly observation intervals, tests of Propositions 1 and 2 reject the Merton model for almost all firms and all debt issues. The differences being attributable to the reduced sample size.

This evidence rejects the simplest form of the structural approach to modeling credit risk. However, the implications for its extensions and generalizations are not studied herein. Perhaps our testing methodology can be extended to study these generalizations as well. This exercise, however, awaits subsequent research.

Appendix – Merton's (1974) formula

Let $\mathrm{d}V_t / V_t = \alpha \mathrm{d}t + \sigma \mathrm{d}W_t$ with W_t a standard Brownian motion. Then,

$$D_t = V_t N(h_1) + F\mathrm{e}^{-\int_t^T r_u \mathrm{d}u} N(h_2)$$

where

$$h_1 = \frac{\ln\left(F\mathrm{e}^{-\int_t^T r_u \mathrm{d}u} \big/ V_t \right) - \frac{1}{2}\sigma^2 (T-t)}{\sigma\sqrt{(T-t)}}$$

$$h_2 = -h_1 \sigma\sqrt{(T-t)}$$

$N(\cdot)$ is the cumulative standard normal distribution function.

We have that: $\partial D_t / \partial V_t = N(h_1) > 0$. Also, $V_t = D_t + E_t$. So, $\partial E_t / \partial V_t = 1 - N(h_1) > 0$.

Proof of Proposition 1 For the proof only, let subscripts denote partial derivatives. Using Ito's lemma, for a time t satisfying hypothesis (v):

$$\mathrm{d}D = D_t \mathrm{d}t + D_V \mathrm{d}V + \tfrac{1}{2} D_{VV} \mathrm{d}V^2$$

$$\mathrm{d}D - rD\mathrm{d}t = -rD\mathrm{d}t + D_t \mathrm{d}t + D_V [\mathrm{d}V - rV\mathrm{d}t] + D_V rV\mathrm{d}t + \tfrac{1}{2} D_{VV} \mathrm{d}V^2$$

Both $\mathrm{d}D - rD\mathrm{d}t$ and $D_V[\mathrm{d}V - rV\mathrm{d}t]$ are martingales using the no arbitrage hypothesis, under the martingale measure, which implies that

$$0 = -rD\mathrm{d}t + D_t\mathrm{d}t + D_V rV\mathrm{d}t + \tfrac{1}{2}D_{VV}\mathrm{d}V^2$$

(This is the Black–Scholes partial differential equation.) This gives:

$$\mathrm{d}D - rD\mathrm{d}t = D_V\left[\mathrm{d}V - rV\mathrm{d}t\right]$$

Similarly,

$$\mathrm{d}E - rE\mathrm{d}t = E_V\left[\mathrm{d}V - rV\mathrm{d}t\right]$$

Hence

$$\mathrm{d}D - rD\mathrm{d}t = \frac{D_V}{E_V}\left[\mathrm{d}E - rE\mathrm{d}t\right]$$

Since $D_V > 0$ and $E_V > 0$ we get our result. Q.E.D.

PROOF OF PROPOSITION 2 By Ito's lemma, we have that:

$$\mathrm{d}y = -\frac{\mathrm{d}D}{D(T-t)} + \left(\tfrac{1}{2}\right)\frac{\mathrm{d}D^2}{D^2} - \frac{\log D}{(T-t)^2}\mathrm{d}t$$

Or, equivalently:

$$\mathrm{d}y = -\frac{\mathrm{d}D - rD\mathrm{d}t}{D(T-t)} + \left(\tfrac{1}{2}\right)\frac{\mathrm{d}D^2}{D^2} + \frac{(y-r)}{(T-t)}\mathrm{d}t$$

Also since $\mathrm{d}r = 0$, $\mathrm{d}(y - r) = \mathrm{d}c$. So,

$$\mathrm{d}c = -\frac{\mathrm{d}D - rD\mathrm{d}t}{D(T-t)} + \left(\tfrac{1}{2}\right)\frac{\mathrm{d}D^2}{D^2} + \frac{c}{(T-t)}\mathrm{d}t$$

Using the result from Proposition 1, we get:

$$\mathrm{d}c = -\frac{D_V}{E_V D(T-t)}\left[\mathrm{d}E - rE\mathrm{d}t\right] + \left(\tfrac{1}{2}\right)\frac{\mathrm{d}D^2}{D^2} + \frac{c}{(T-t)}\mathrm{d}t$$

Note that

$$\left(\tfrac{1}{2}\right)\frac{\mathrm{d}D_t^2}{D_t^2} + \frac{c_t}{(T-t)}\mathrm{d}t > 0$$

This yields the following test: If $\mathrm{sign}(\mathrm{d}E - rE\mathrm{d}t) < 0$, then $\mathrm{sign}(\mathrm{d}c) > 0$. If $\mathrm{sign}(\mathrm{d}E - rE\mathrm{d}t) > 0$, then $\mathrm{sign}(\mathrm{d}c)$ is ambiguous. It depends on size of the positive stock price increase. Q.E.D.

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