Risk Management for Alternative Investments

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Introduction

This chapter provides an overview of financial risk management for alternative investments (AI). These investment products differ from traditional investments, such as stocks, bonds, and cash. AI include hedge funds, commodities, real estate, and private equity. Alternatives are often viewed as having relatively low correlations with traditional asset classes, which should provide diversification benefits to the investor's portfolio. They have generally provided good returns with limited volatility or correlation relative to traditional investments. As a result, they are becoming increasingly important in investors' portfolios.

Alternatives pose special problems for risk management. They cover a broad range of investment styles. At one end are hedge funds or commodity trading advisors (CTAs) that trade actively, generally using liquid instruments. At the other extreme are funds such as private equity where positions are kept for years in assets that are not marked to market.

In each case, risk management is a challenge. Funds that trade actively require a position-based risk management system to monitor and manage their rapidly changing risks. The problem is opposite for funds that invest in illiquid assets. Illiquidity implies that prices do not change often, which makes it difficult to assess valuation properly, let alone risk.

AI managers generally take views on markets and securities. This process should add value to the investment for a number of reasons. Firstly, AI managers have much wider investment opportunities and are less regulated than managers in traditional asset classes. They can short securities, leverage their portfolio, use derivatives, and generally invest across a broader pool of assets. They can set performance fees; they can impose lockup and minimum redemption notice periods; they do not have to disclose their holdings publicly. Secondly, AI managers have a stronger financial motivation to perform because of the compensation structure of the industry. Managers receive not only a fixed annual management fee ranging from 1% to 2% of assets under management (AUM) but also an incentive fee that typically represents 20% of the annual profits. This helps align the managers' incentives with investors. The prospect of such riches undoubtedly attracts many of the best minds in the business. In the hedge fund industry, Agarwal et al. (2009) indeed find that greater managerial discretion and managerial incentives are associated with superior performance.

The very features that generate superior performance, however, should cause serious concerns to investors. Alternatives managers can be secretive about their strategy and positions. They have more latitude in setting their net asset value (NAV) than regulated entities. So, there is a possibility of fraud or undue risk exposures going undetected that could lead to blowups. In particular, incentive fees may tempt the manager to increase risks.

To some extent, risks can be mitigated if the portfolio managers have invested a substantial fraction of their wealth into the fund itself. For leveraged funds, risk can also be monitored by lenders, such as prime brokers for hedge funds. The prime broker, however, is mainly concerned about losses it could incur if the hedge fund defaults and not necessarily about losses to investors. So, risk monitoring by the prime broker may not be sufficient. This is why risk management is particularly important for the alternatives industry. Yet it is also more difficult than for traditional asset classes.

The purpose of this chapter is to provide an overview of risk management techniques for the alternatives industry. The emphasis is on **market risk**, which is the risk of losses due to movements in financial market prices or volatilities. In investment portfolios, this also includes **credit risk**, as changes in perceived default probabilities or actual defaults are incorporated into market prices. **Liquidity risk**, which is the risk of losses due to the need to liquidate positions to meet funding requirements, is also discussed. Investments in alternatives also involve operational and business risks, however, which are not considered here.

This chapter is structured as follows. Section 1 describes the general design of risk measurement systems, which are constructed from positions, risk factors, and a risk engine. It compares the pros and cons of position-based and returns-based risk measures. The process of mapping position on risk factors reveals exposures, which are presented in Section 2. The section also reviews conventional risk measures such as leverage and concentration. Section 3 then discusses how to summarize the distribution of a single position or top-level portfolio distribution, comparing various aggregate measures of downside risk such as standard deviation and Value at Risk (VAR). Next, Section 4 presents an overview of the different approaches to VAR models, including the delta-normal approach, historical simulation, and Monte Carlo simulation. It also shows how risk systems can be easily extended to stress tests and can be used to manage risk by drilling down into its components. Section 5 discusses problems created by illiquidity for risk measures. Illiquidity causes serious biases in measures of volatility and correlations with other asset classes. Section 6 discusses limitations from traditional risk measurement systems. Section 7 discusses problems posed by the lack of transparency for some alternatives investments and proposes solutions. Finally, Section 8 concludes.

1. Risk Measurement Systems

Ideally, market risk should be measured using a position-based risk measurement system, which is described in Figure 1. This involves several steps. First, the risk manager must collect all the current **positions** in the portfolio and map them on the market risk factors, via factor **exposures.** Second, the risk manager must construct the statistical distribution of **risk factors** from market data. Third, the risk manager must use the **risk engine** to derive the distribution of profits and losses on the portfolio. This can be summarized by several measures, such as the worst loss at a specified confidence level, also called Value at Risk (VAR).

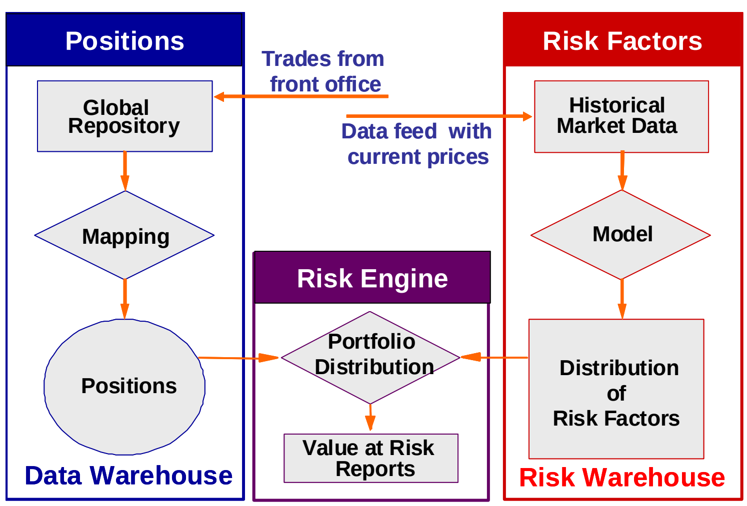

Fig. 1. Components of a Risk Measurement System

The key feature of this system is that it is **position-based**. Traditionally, risk has been measured from **returns-based information**, i.e., from the time-series of historical returns on the portfolio. On the one hand, a returns-based risk system is easy and cheap to implement. On the other hand, returns-based measures suffer from severe drawbacks. They offer no information for new instruments and markets. They are completely ineffective for emerging managers or funds that have short track records. Such managers, however, account for a large fraction of the alternatives universe. Returns-based measures do not capture—or rather, are very slow at identifying—style drift. They may not reveal hidden risks.

As an example of this important issue, Lo (2001) considers a hypothetical fund, called Capital Decimation Partners, which seems to perform very well. Based on historical returns, the fund has a high **Sharpe ratio**, defined as the ratio of excess average return to volatility. It turns out, however, that the fund follows a very simple trading strategy, which is to sell out-of-the-money put options on the S&P index. As long as the options are not exercised, the portfolio generates positive and steady returns, which reflect the option premium. On rare occasions, however, the fund could suffer extreme losses. In this case, the returns-based volatility is totally misleading. More generally, returns-based risk measures give no insight into the real risk drivers of portfolio strategy.

Most of these drawbacks are addressed by position-based risk measures. They can be applied to new instruments, markets, and managers. These use the most current position information, which should reveal style drift or hidden risks. For example, Jorion (2007) shows that the risk of Capital Decimation Partners can be captured and controlled effectively by position-based risk systems. In addition, position-based systems can be used for forward-looking stress tests.

Position-based risk systems, however, can be challenging to implement and have drawbacks that risk managers must understand. First, they require more resources and are expensive to implement. A large bank could have several million positions, in which case aggregation at the top level is a major technology challenge. Second, position-based risk measures assume that the portfolio is frozen over the time horizon considered. Taking one month as an example, these risk measures combine the fixed portfolio positions at the beginning of the month with risk factor returns over the month, thus ignoring any active trading that would take place in practice. To some extent, this problem can be mitigated by more frequent risk measurement. Finally, position-based systems are susceptible to errors and approximations in data and models. They require modeling all positions from the ground up, repricing instruments as a function of movements in the risk factors. The modeling of some instruments can be complex, leading to **model risk**. Even so, position-based risk measures are vastly more informative than returns-based risk measures. This explains why modern risk management systems are built from position-level information.

2. Conventional Risk Measures

2.1 Factor Exposure Measures

Exposures are a major component of position-based risk measurement systems. Their advantage is that they do not consider the range of potential movements in the risk factors and thus do not require assumptions about statistical distributions. This is also a drawback, as exposure measures are factor-specific and do not aggregate across different types of factors. There is no way, for instance, to combine the duration of the bonds in a portfolio with the beta of its stocks for an overall risk measure. Nevertheless, exposures are intuitive to understand and are widely used in risk management and reporting.

Exposures are related to the mapping procedure for positions in Figure 1. Mapping is the process of replacing positions by dollar exposures on the risk factors. Consider for example a position in a default-free fixed-coupon bond, such as a US Treasury bond. The most important risk factor for this bond is the movement in risk-free yields. Initially, assume that the yield curve is flat and moves in a parallel fashion. For each position, the exposure to this risk factor can be represented by **modified duration *D****. This is constructed from information about the bond's cash flows and the sequencing of payments. The relative change in the market value of the position P can be explained by the following combination of this duration and the movement in the risk factor Δy

$$\frac{\Delta P}{P} = -D^{*}\ \Delta y \tag{1}$$

This first-order, linear approximation can also be rewritten in terms of dollar duration (D^*P). In the mapping process, the position in the bond can be replaced by its dollar duration:

$$\Delta P = -(D^{*}P)\ \Delta y \tag{2}$$

If all N bonds in the portfolio are exposed to the same risk factor, then duration can be aggregated at the top level of the portfolio using the market weights of all positions w_i

$$D_P^{*} = \sum_{i=1}^{N} w_i D_i^{*} \tag{3}$$

The same principle applies to other measures of exposure, which are listed in Table 1.

Table 1. Measures of Exposure

Risk Factor	**Exposure**
Movements in equity index price	Beta
Movements in the risk-free rate	Duration
Quadratic move in rates	Convexity
Movements in credit spreads	Spread duration
Movements in the risk factor	Delta
Quadratic move in risk factor	Gamma
Implied volatility	Vega
Default	Jump to recovery

These exposures are particularly important to monitor for major market risk factors, such as movements in the general level of equities, movements in risk-free interest rates, and movements in credit spreads. As Equation (3) indicates, exposures are additive across the entire portfolio. As a result, they do not diversify away as the number of positions increases in the portfolio.

This point can be demonstrated by considering a portfolio of N stocks, where returns are driven by a general equity index R_M plus residual effects ε which, as a first approximation, are assumed independent across stocks:

$$R_i = \beta_i R_M + \varepsilon_i \tag{4}$$

The return of a stock portfolio can be written as

$$R_P = \sum_{i=1}^{N} w_i R_i = \sum_{i=1}^{N} w_i \beta_i R_M + \sum_{i=1}^{N} w_i \varepsilon_i = \beta_P R_M + \sum_{i=1}^{N} w_i \varepsilon_i \tag{5}$$

where β_P is the portfolio beta. As a result, the variance can be decomposed into two terms:[1]

$$V(R_P) = \beta_P^2 V(R_M) + \sum_{i=1}^{N} w_i^2 V(\varepsilon_i) \tag{6}$$

As the portfolio becomes more diversified, the second term becomes smaller.[2] In contrast, the first term depends on the average portfolio beta and the variance of the market factor only. Because the average beta does not depend on the number of positions, it is not a diversifiable exposure. This is why institutional investors, who typically have large direct allocations to equities, should also monitor the beta exposure of their alternative investments to be aware of their total exposure to equities.

While useful, these measures of exposure have limitations. Linear exposures do not account for large movements in the risk factors. Quadratic measures improve the approximation but only to some extent. In addition, exposures do not aggregate across risk factors, which is why statistical risk measures are also needed.

2.2 Portfolio Exposure Measures

Conventional portfolio exposure measures provide very simple indicators of total risk. The most common family of measures is based on leverage. Consider for instance a stock-only hedge fund with the balance sheet described in Table 2. The fund starts with $100 in equity, borrows $20 from the broker and purchases $120 in some stocks. The fund then borrows and short-sells $80 worth of other stocks.

Table 2. Hypothetical Hedge Fund Balance Sheet

Assets		Liabilities	
$120	long stock	$80	short stock
$80	cash lent to stock owner	$20	loan
		$100	equity

Define now V_A, V_L, V_S, and V_E as the market value of total assets, long stock positions, short stock positions, and of the equity, respectively (in absolute values). For a regular corporation, balance-sheet leverage is conventionally measured by V_A / V_E. For investment funds, cash assets and liabilities are ignored. The usual measures of leverage are:

- Long leverage, or V_L / V_E
- Short leverage, or V_S / V_E
- Gross leverage, or $(V_L + V_S) / V_E$
- Net leverage, or $(V_L - V_S) / V_E$

Each of these measures has a different use and interpretation. Generally, higher leverage indicates higher risk. Long leverage, for instance, is the inverse of the drop in the value of the

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[1] Note that there is no covariance term between the market and residual effects, because these are independent by virtue of the regression framework. Also, there are no covariance terms between residual effects because these are assumed independent to each other.

[2] This can be proved in the simple case where all weights are the same $w=1/N$ and all the residual variances are equal. As N increases, the second term then becomes $\sum_{i=1}^{N} w_i^2 V(\varepsilon_i) = \sum_{i=1}^{N} (1/N)^2 V(\varepsilon) = N(1/N)^2 V(\varepsilon) = (1/N) V(\varepsilon)$, which goes to zero as the number of stocks N increases. So, residual risk is diversifiable, unlike market exposure.

long positions that would wipe out the equity, assuming other positions are not changed. In this case, long leverage is $120/$100=1.2. Hence, if the long positions were to fall by 1/1.2=83.33%, the portfolio would lose $120×83.33%=$100, which would wipe out the equity of $100. Similarly, short leverage is $80/$100=0.8, meaning that if the short positions went up by 1/0.8=125%, the equity would be wiped out. In this case, the portfolio would lose $80×125%=$100.

An even worse scenario considers the gross leverage, which is (120+80)/100=2.0 in this case. Disaster would happen if the longs were to go down by 50% and the shorts up by 50%. Of course, it is highly unlikely that both the long and the short positions would go in the worst possible direction at the same time. Net leverage, which is (120-80)/100=0.4 is more meaningful for this reason. It means that the equity would be wiped out if both longs and shorts went down by 1/0.4, or 250%. The loss in this case would be $120×250%-$80×250%=$100.

The advantage of these measures is that they can be constructed from portfolio listing information. The disadvantage, however, is that they are based on simplistic assumptions, which is that all positions among assets and/or liabilities move by the same amount. This may be acceptable for all-equity portfolios, but certainly less so with fixed-income products. For the latter, market values can be adjusted to 10-year equivalents. In addition, these leverage measures do not consider off-balance sheet items nor the quality of financing.

Other measures of risk involve classifying the market value of the portfolio into different categories: asset class, industry concentration, region, issuer market capitalization, issuer style (e.g., value or growth), debt credit rating, debt duration, and so on. These are simple measures of diversification. Measures of concentration can be also reported, such as the list of positions with the largest long and short market values.

3. Statistical Risk Measures: Single Investment or Portfolio

This section illustrates how to compute measures of market risk for a single investment, or at the top level of an investment portfolio. Consider for example a hedge fund trader with a position in a foreign currency, say $4 billion short the yen against the dollar. How can we describe the potential loss on this position over the next day?

This example is particularly appropriate because the risk factor, the yen/dollar exchange rate, is priced in a liquid market for which there is a long history that spans quiet and turbulent times. Hence, historical data should be a good guide from which to build the statistical distribution of future risks.

3.1. Building the Distribution

To answer this question, we use 10 years of historical daily data on the yen/dollar rate (1999-2008) and simulate a daily return. The simulated daily return in dollars is then

$$R_t(\$) = -Q_0(\$)\ [S_t - S_{t-1}]/S_{t-1}$$

where Q_0 is the current dollar value of the position and S_t is the spot rate in dollar per yen. For instance, the exchange rates on December 31, 1998 and January 4, 1999 are 112.80 and 111.65 yen/$, respectively. As the usual convention in this market is to quote the exchange rate in yen, we need to invert it to measure dollar values. The simulated return is then $R_2(\$)$ = –$4 billion (1/111.65)-(1/112.80)]/(1/112.80) = –$4 billion x 1.03% = –$41.2 million. Repeating this operation over the entire sample, or N=2,539 trading days, creates a time-series of fictitious returns, which is plotted in Figure 2.

This approach is position-based because it uses the most current position, which is

Q_0. In contrast, a returns-based approach would use the history of profits and losses (P&L) for the trader. This is largely irrelevant, however, if the trader changes the portfolio substantially.

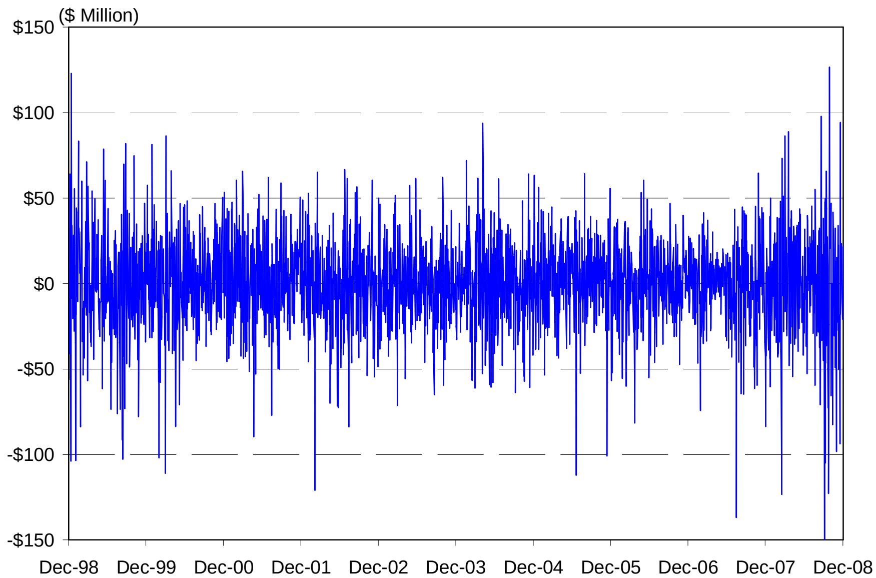

Fig. 2. Time-Series of Simulated Daily Returns on Portfolio ($ Millions) (Source: Author's Computations)

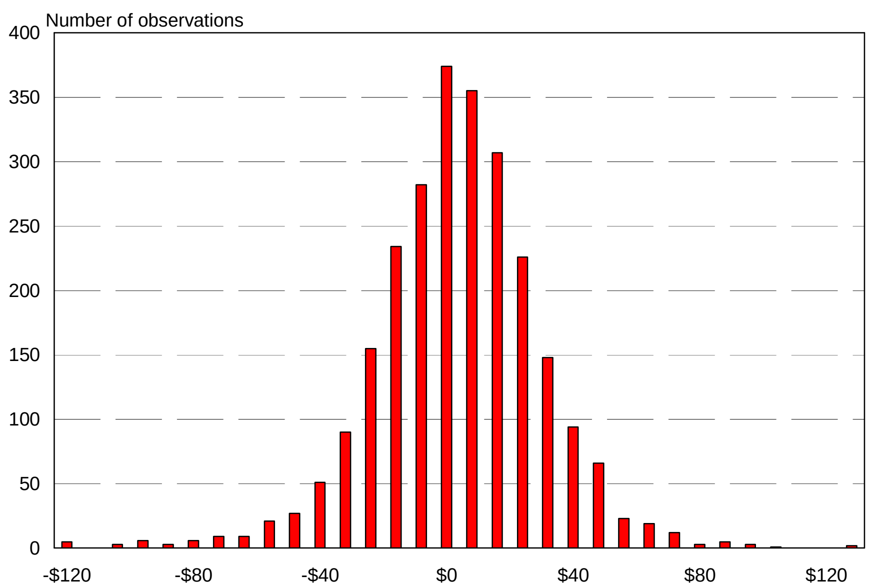

Fig. 3. Distribution of Simulated Daily Returns on Portfolio ($ Millions)

The statistical distribution of P&L can be summarized by a **histogram**, which compiles the number of observations within ranges and is shown in Figure 3. For example, there are five cases of a loss worse than –$120 million, none between –$120 and –$115, and so on. This entire distribution should be of interest to the risk manager. Generally, this can be described by the probability density function, or pdf, $f(x)$.

3.2. Summarizing the Distribution

Single summary statistics usefully describe the distribution of profits and losses. Define x_i as the value of an observation, and N as the number of observations. The **mean** μ is the first moment, or expectation of X

$$E(X) = \frac{1}{N}\sum_{i=1}^{N} x_i \tag{7}$$

In this case, the mean is -$0.43 million. As we shall see, this is small compared to typical risk measures.

The dispersion can be assessed by the **standard deviation** (SD), usually defined as σ. This is constructed from the variance or second moment as

$$\mathrm{SD}(X) = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}[x_i - E(X)]^2} \tag{8}$$

In this example, the standard deviation, or volatility of returns is $26.6 million. Distributions with greater volatility are more risky. This measure, however, is symmetric and treats equally positive and negative observations of like size. Another measure that focuses on the downside risk is the semi-standard deviation. Define N_L as the number of points below zero. The risk measure is

$$\mathrm{SD}_L(X) = \sqrt{\frac{1}{N_L}\sum_{i=1}^{N}[\mathrm{Min}(x_i, 0)]^2} \tag{9}$$

In this case, the measure is $28.1 million, slightly greater than the standard deviation. This suggests that the distribution has a longer tail on the downside than on the upside.

Symmetry can be summarized by the **skewness** coefficient, which is the scaled third moment S. This is the expectation of the deviation from the mean to the third power

$$S = \frac{E\left[(x-\mu)^3\right]}{\sigma^3} \tag{10}$$

Negative skewness indicates a long left tail, or the possibility of larger losses than gains. In our example, the skewness is −0.45, which is slightly negative. Generally, a skewness coefficient below −1 should be source for concern.

The size of the tails can be assessed by the **excess kurtosis** coefficient K, which is the scaled fourth moment in excess of 3, or

$$K = \frac{E\left[(x-\mu)^4\right]}{\sigma^4} - 3 \tag{11}$$

An excess kurtosis greater than 0 indicates that the distribution has fatter tails than a normal distribution and hence may generate more extreme values. In our example, the excess kurtosis is 3.34, which reveals much fatter tails than in a normal distribution. This is indeed typical of most financial series. Generally, an excess kurtosis coefficient above 2 should be source for concern.

Another measure of downside risk is the lower **quantile**, which is the cutoff value q that corresponds to a prespecified confidence level c

$$P(X \geq q) \geq c \tag{12}$$

Note that this is defined in terms of the cumulative probability to the right of q. Equivalently, the cumulative probability to its left is $1-c$.

The quantile is usually transformed into a positive number that represents a loss, expressed in dollars or whichever currency is used. This is also known as **Value at Risk** (VAR), or the worst loss such that there is a low, prespecified probability that the actual loss will be larger, VAR = $-q$. For example, suppose that we pick a 95% confidence level. We first compute the number of observations required in the left tail from (1-c) N = 5% ×2,543 =126.95. We then sort observations from the lowest return to the highest. Starting from the bottom, the observations ranked 126 and 127 are -$42.41 and -$42.40, respectively, with frequencies of 4.963% and 5.002%. Hence, q=–$42.41, and VAR is $42.41 million.[3] The risk manager can then give the following economic interpretation to this number: "Under normal market conditions, the most the portfolio can lose over one day is about $42 million at the 95 percent confidence level."

VAR has become widely used as a statistical measure of portfolio risk. Notably, it has been endorsed by the Basel Committee (1996) as the basis for the market risk charge for commercial banks.[4] This is the amount of capital that the bank must keep on its books as a buffer against trading losses. The advantage of VAR is that it takes into account the shape of the distribution function. Negative skewness or high kurtosis will be reflected in VAR.

A disadvantage of VAR, however, is that it sheds no light on the size of losses once VAR is exceeded. A complementary risk measure is the **conditional VAR (CVAR)**, which is the average of losses beyond VAR. Using the ranked observations, we have M losses up to VAR. The CVAR is then

$$\text{CVAR} = \frac{1}{M}\sum_{i=1}^{M}(-x_i) \qquad (13)$$

Figure 4 displays the VAR and CVAR risk measures for this sample. Here, CVAR is $63.6 million.

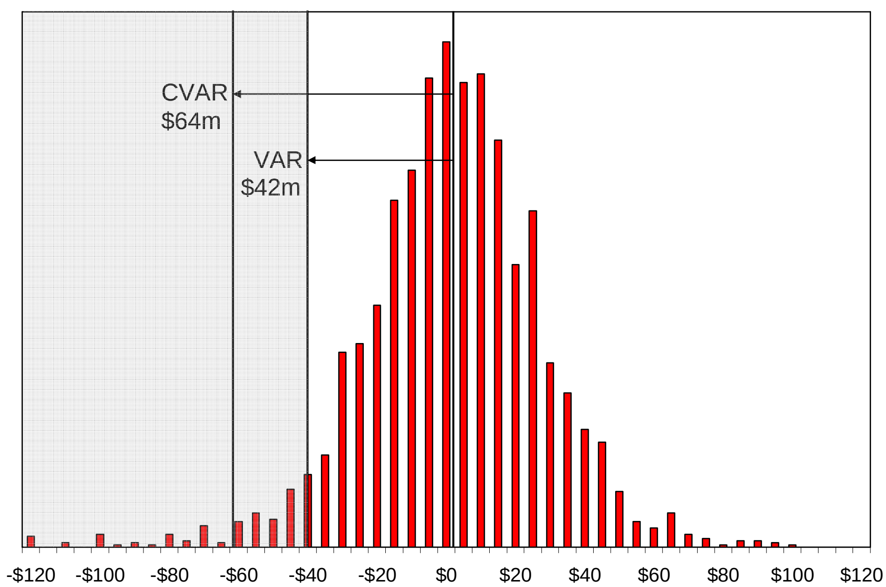

Fig. 4. Risk Measures for the Empirical Distribution ($ Millions)

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[3] Sometimes, this number is expressed in terms of deviations from the mean. In this example, the mean is close to zero, and as a result, the VAR number would not change much.

[4] The Basel Committee on Banking Supervision consists of central bankers from the Group of Ten (G-10) countries. It prescribes minimum standards to regulate internationally active commercial banks.

By construction, this must be greater than VAR. Generally, the two numbers are similar in terms of order of magnitude. In this case, the CVAR is 50% greater than the VAR of $42 million. A portfolio could contain short positions in out-of-the-money options that could lose a lot of money if exercised. If this were the case, CVAR could be several times VAR. This raises a red flag that the portfolio is exposed to extreme risks.

Finally, it should be noted that even CVAR does not characterize the absolute worst loss. This is basically impossible to ascertain if movements in risk factors are unbounded.

3.3. Parametric vs. Non-Parametric Approaches

The risk manager, however, may decide that the distribution of returns could be well described by a **parametric** distribution, such as the normal distribution. This considerably simplifies the analysis because the distribution is then characterized solely by two parameters, its mean μ and standard deviation σ. The quantile around the mean becomes a multiple of σ, using a multiplier α that depends on the confidence level. For example, if Z has a standard normal distribution and c = 95%, we know from statistical tables that $P(Z \geq -1.645) = 95\%$,[5] so that α = 1.645. Hence, VAR can be defined as

$$\text{VAR} = \alpha\sigma \tag{14}$$

where σ is measured in dollar terms. This considers risk in terms of the deviation from the mean of the distribution on the target date. Another approach is to define risk in terms of changes from the initial portfolio value, in which case the formula for VAR should adjust for the mean, $\text{VAR} = \alpha\sigma - \mu$. However, it is common to ignore the mean for two reasons. First, when the estimation interval is small (i.e., daily), μ is generally small, in which case it would be sensible to set it to zero. Second, estimates of μ are less accurate over short horizons, which implies that the estimated value of μ is typically not statistically different from zero.

If σ is measured in terms of rates of return, it should be multiplied by the current value of the portfolio W, giving VAR = $\alpha\sigma W$. In our example, VAR = 1.645×0.664%×$4 billion = 1.645×$26.6 = $43.7 million.[6] Note that is close to the empirical, non-parametric VAR of $42.4 million. At higher confidence levels, however, these two numbers start to diverge from each other because actual distributions have fatter tails than the normal.

Figure 5 displays the fitted normal distribution. Note that, relative to Figure 3, the tails are much thinner. This confirms the previous observation that the empirical kurtosis of the data is greater than that of a normal distribution.

[5] Note that 1.645 is the standard normal deviate for a one-tailed probability of 95%. For a normal distribution, the deviate for a two-tailed probability of 95% is 1.96. This is because 2.5% of the distribution is below -1.96 and 2.5% is above +1.96. So, the usual association of α around 2 for a 95% confidence level corresponds to a two-tailed test.

[6] Note that the number 0.664% is the previously reported standard deviation of returns of $26.6 million expressed as a percentage of the portfolio value o $4 billion.

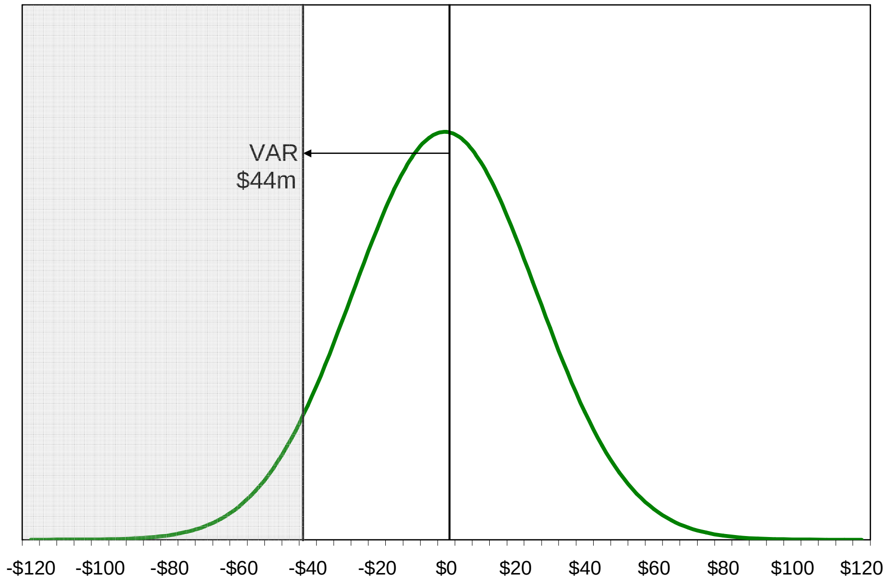

Fig. 5. Risk Measures for the Normal Distribution ($ Millions)

Table 3 reports the quantiles α of a standardized normal distribution. For a confidence level of 95%, for example, the multiplier α is 1.645. Table 3 also reports the multiplier corresponding to the conditional VAR. For a confidence level of 95%, this is 2.063. By construction, this number must be greater than α. The two numbers, however, are similar in magnitude. Here, CVAR is 25% greater than VAR.

Table 3. Lower Quantiles of the Standardized Normal Distribution

	Confidence level (%)						
	99.99	99.9	99.0	97.5	95.0	90.0	50.0
Quantile $(-\alpha)$	-3.719	-3.090	-2.326	-1.960	-1.645	-1.282	-0.000
$E(Z\|Z<-\alpha)$	-3.957	-3.367	-2.665	-2.338	-2.063	-1.755	-0.798

If the risk manager believes the distribution of the variable under consideration is substantially different from a normal distribution (e.g., has fatter tails), the manager could use the multiplier for another parametric distribution, such as the student *t*.[7] In this case, the multiplier α will be higher. More generally, the first three or four moments can be used to adjust the normal quantile using the **Cornish-Fisher expansion**. The Cornish-Fisher expansion is a method that allows us to estimate quantiles of an arbitrary distribution from its moments. We illustrate this method with the first three moments, up to the skewness *S*. The Cornish-Fisher expansion is

$$VAR = \alpha'\sigma \tag{15}$$

Here α' is related to the original α according to the following relationship:

[7] The student *t* is a symmetric probability distribution where the thickness of tails depends on a parameter ν called degrees of freedom. As ν tends to infinity, the distribution tends to the normal pdf. As ν decreases, the distribution has increasingly fatter tails. Thus the parameter can be chosen to fit the empirical data.

$$\alpha' = \alpha + \frac{1}{6}\left(\alpha^2 - 1\right) S \tag{16}$$

As an example, with $S = -0.5$, the coefficient at the 95% level of confidence is increased from 1.645 to 1.787. More negative skewness indeed means that the distribution is more risky. With a normal disribution, $S = 0$ and α remain as 1.645 as expected.[8]

The parametric approach must be more efficient than a non-parametric approach because it makes a strong assumption about the shape of the distribution (provided the assumption is correct). In contrast, a non-parametric approach makes no such hypothesis—other than assuming than the past is representative of the future.

The increase in the VAR precision can be traced to the fact that the computation of the standard deviation uses all the data points in the sample and, as a result, is estimated rather precisely. In contrast, the quantile only uses the values of the two numbers around the cutoff point. As a result, the sample quantile is much less precisely estimated, or has substantial **estimation error**. In other words, another data sample could yield a totally different number, especially if the confidence level is high. When VAR is estimated from the standard deviation, it is much less susceptible to variations in the data. Therefore, the parametric method is more efficient. This reflects the general principle in statistics that putting more structure on a model will give more precise results, provided that the assumptions are valid.

3.4. Choice of Horizon and Confidence Level

To measure risk, we need to define (1) the horizon and, for VAR-type measures, (2) the confidence level. Consider first the choice of the horizon. For trading portfolios, this is typically short-term, such as one day. For investment portfolios, the horizon is longer, typically from one month to one year.

Longer horizons increase risk measures. This can be shown in the case where returns are identically and independently distributed across subperiods. Consider for example, our return on the short yen position from before, but during two consecutive days. If we measure returns in logarithmic form, the two-day return is $R_{12} = \ln(S_2 / S_0) = \ln(S_1 / S_0) + \ln(S_2 / S_1) = R_1 + R_2$. The variance is

$$\sigma_{12}^2 = \sigma_1^2 + \sigma_2^2 + 2 \ \mathrm{Cov}(R_1, R_2) \tag{17}$$

If returns are independent from one day to the next, the covariance term is zero. If distributions are identical, we have $\sigma_2^2 = \sigma_1^2$ and the 2-day variance reduces to $\sigma_{12}^2 = 2\sigma_1^2$. This shows that the variance increases linearly with time and thus the volatility increases with the square root of time. More generally, defining T as the number of days, we have

$$\sigma_T = \sigma_1 \sqrt{T} \tag{18}$$

The same adjustment applies to VAR when daily returns have normal distributions, because a linear combination of jointly normal variables is itself normal. As a result, both sides of Equation (18) can be multiplied by α, which gives the square root of time rule:

$$\mathrm{VAR}_T = \mathrm{VAR}_1 \sqrt{T} \tag{19}$$

For instance, in our hedge fund case, the daily VAR was \$43.7 million, assuming a normal density. Extrapolating to one month, or 21 trading days, gives $\$43.7\ \sqrt{21} = \$200.1$

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[8] The expanded form of the Cornish-Fisher formula calculates VAR using kurtosis as well.

million. Note that this assumes that daily returns are uncorrelated. In the case of the yen/dollar rate from before, this is indeed verified because the first-order correlation, or **autocorrelation**, coefficient is -0.034 only, with a standard error of 0.020. The *t*-statistic is small, at $t = -0.034/0.020 = -1.7$, indicating absence of statistical significance. This suggests that returns for one day are not useful to forecast returns the next day. Later, we will see that this assumption of zero autocorrelation does not hold well for less liquid investments.

The choice of the horizon depends on the use of VAR. If the goal is to provide an accurate measure of downside risk, the horizon should be relatively short, ideally less than the average period for major portfolio rebalancing. In contrast, if the VAR number is being used to decide how much capital to set aside to avoid bankruptcy, then a long horizon is advisable. This is because institutions need to have enough time for corrective action as problems start to develop. The Basel rules require a 10-day horizon for market risk and annual horizon for credit and operational risk.

Next, we turn to the choice of the confidence level. The higher is the confidence level, the greater the VAR measure. Assuming a normal distribution, we can use the quantiles in Table 1 to adjust the 95% VAR to a different confidence level. For example, the 99% VAR would be \$43.7 times (2.326/1.645), or \$61.8 million. From the empirical distribution, the non-parametric VAR is \$75.3 million. In this case, the normal-based VAR understates the empirical VAR.

As with the horizon, the choice of the confidence level depends on the use of VAR. If the goal is to provide a general measure of downside risk, the confidence level should not be too high, typically 95% or 99% as required by the Basel Committee. Here, what really matters is consistency of the VAR confidence level across trading desks or across time. In contrast, if the VAR number is being used to decide how much capital to set aside to avoid bankruptcy, then a high confidence level is advisable to keep the fund safe. The Basel rules require a 99.9% confidence level for credit and operational risk.

Institutions now routinely report measures of **economic capital**, which is the amount of capital an institution would voluntarily set aside to support its business activities. This is typically estimated as a VAR measure derived from the distribution of total profits and losses at a very high confidence level such as 99.97% over a year. This approach is fraught with problems, however. The first one is that the institution must take into account all of its risks and measure their distribution properly. The second is that very high confidence levels make it very difficult to estimate VAR measures precisely. This is because there are few if any observations in the left tail.[9]

Long-Term Capital Management (LTCM) is an example of a fund that blew up because it did not have enough capital.[10] At the beginning of 1998, LTCM thought that \$4.7 billion of capital was more than sufficient to absorb a worst-case situation. By August, the fund had lost \$2.4 billion. It was unable to raise additional funds and materially changed its risk profile. By September 23, the fund had lost another \$2 billion, forcing a Fed-orchestrated bailout. The portfolio managers had badly underestimated how much they could lose.

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[9] See Jorion (1996) for an analysis of the estimation error in risk numbers. Rebonato (2007) provides a lucid criticism of economic capital measures, which he calls “science fiction” numbers. For instance, assessing an empirical VAR measure at the 99.97% level of confidence would require 3 observations in the left tail out of 10,000 annual observations.

[10] Jorion (2000) provides a risk management perspective of LTCM.

3.5 Backtesting

No risk measurement system would be complete without a process for backtesting. This involves systematic comparisons of the actual returns with the risk forecasts. With a well-calibrated system, the number of losses worse than VAR, also called exceptions, should correspond closely to the confidence level. For example, backtests of a 1-day VAR at the 99 percent level of confidence over a period of one year should yield, on average, 2 to 3 exceptions per year (more precisely, 1% times 252 trading days in a year, or about 2.5 observations). Too many exceptions should cause the risk manager to re-examine the models.

To implement backtests, the risk manager needs to construct a decision rule. Say that the cutoff point is 4 exceptions, beyond which the model is deemed to have failed. This involves two opposite tradeoffs. The first is the probability of rejecting a correct model, which is called a Type I error. The second is the probability of not rejecting a false model, which is called a Type II error. Type I errors occur due to bad luck, perhaps unusually volatile markets. Type II errors occur when poor models are not detected. Increasing the cutoff point from 4 to 5 will decrease the Type I error rate, but increase the Type II error rate.

The Basel Committee has setup a simple system for verifying the risk numbers reported by banks, with a "green" zone for up to 4 exceptions, a "yellow" zone for 5 to 9, and a "red" zone for 10 and above. One can show that, assuming that the model is correctly specified, the probability of observing 5 or more exceptions is 10.8%, which is the Type I error rate.

More generally, a simple decision rule can be constructed as follows. Define x as the observed number of exceptions over the last T observations. If the VAR confidence level is $c=1-p$, we should expect to see pT exceptions on average. Then compute the statistic

$$z = \frac{x - pT}{\sqrt{p(1-p)T}} \qquad (20)$$

This is approximately distributed as a standard normal variable. Hence, if z is too large, e.g. above 2, the model failed, with a Type I error rate of about 5%. The onus is then on the risk manager to understand why this has happened and how to improve the model.

3.6 Modeling Changes in Volatility

As we saw above, the estimate of volatility is a critical input for calculating VAR and other risk measures. Volatility can change over time, however, which needs to be monitored. Suppose we observe N daily observations on the rate of return r of an asset and we wish to forecast the variance over the next day t The conventional method for computing the variance is, from Equation (8)

$$\sigma_t^2 = \frac{1}{N-1}\sum_{i=1}^{N}\left(r_{t-i} - \mu\right)^2 \qquad (21)$$

In this expression, all observations have the same weight. If the financial environment changes, however, it is more appropriate to assign relatively higher weights to the most recent observations. A popular approach to such weighting scheme is the **Exponentially Weighted Moving Average** (EWMA) model, where the variance forecast is

$$\sigma_t^2 = \left(1-\lambda\right)\sum_{i=1}^{N}\lambda^{i-1}(r_{t-i} - \mu)^2 + \lambda^N \sigma_{t-N}^2 \qquad (22)$$

Here, λ must be assigned a value between 0 and 1. This **decay factor** determines the pattern of weights, which decrease as the observation gets older. If N is large enough, the last term, $\lambda^N \sigma_{t-N}^2$, will be negligible. The EWMA volatility is typically written in the recursive form

$$\sigma_t^2 = (1-\lambda)(r_{t-1}-\mu)^2 + \lambda\sigma_{t-1}^2 \tag{23}$$

Hence, the variance forecast is a weighted average of the recent innovation squared and of the previous day's variance.

As an example, suppose $\lambda = 0.94$ and that the latest volatility forecast is 1%. Assume that we observe a change in price, away from the mean, of 3%. The new volatility forecast is then

$$\sigma_t^2 = (1-0.94)\times 0.03^2 + 0.94\times 0.01^2 = 0.0001480$$

$$\sigma_t = \sqrt{0.0001480} = 1.22\%$$

This shows that a shock of a size greater than the current volatility of 1% pushes up the volatility forecast from 1% to 1.22%. The extent of this effect depends on the decay factor λ. A lower value assigns more weight to recent observations.[11]

The EWMA model is a special case of class of volatility models known as Generalized Autoregressive Conditional Heteroskedastic (GARCH). In the GARCH(1,1) model, the day *t* forecast includes one lag of the innovation and one lag of the variance

$$\sigma_t^2 = \omega + \alpha\times(r_{t-1}-\mu)^2 + \beta\times\sigma_{t-1}^2 \tag{24}$$

The EWMA is a special case of this model where $\omega = 0$, $\alpha = 1-\lambda$, $\beta = \lambda$. In the GARCH model, the constant can be interpreted as a long-run variance $\bar{\sigma}^2$ times $1-\alpha-\beta$. In addition, the GARCH (1,1) model does not force α and β to sum to one, which generates more realistic dynamics in the variance forecast. The GARCH forecast is a weighted average of the long-run variance, of the squared innovation, and of the previous variance.

To illustrate, suppose a GARCH(1,1) is estimated with the following parameters

$$\sigma_t^2 = 0.000006 + 0.05\times(r_{t-1}^2-\mu)^2 + 0.90\times\sigma_{t-1}^2$$

As in the previous example, suppose that the current standard deviation is 1% and the current excess return is 3%. The estimated volatility for day *t* will be

$$\sigma_t^2 = 0.000006 + 0.05\times 0.03^2 + 0.90\times 0.01^2 = 0.0001410$$

$$\sigma_t = \sqrt{0.0001410} = 1.19\%$$

Further, these coefficients imply a long-run volatility of

$$\bar{\sigma}^2 = \frac{\omega}{(1-\alpha-\beta)} = \frac{0.000006}{(1-0.05-0.90)} = 0.000120$$

$$\bar{\sigma} = \sqrt{0.000120} = 1.10\%$$

In summary, these models adapt to changing financial environments and allow more responsive measures of risk.

[11] RiskMetrics uses an EWMA model with $\lambda = 0.94$ to estimate the daily volatility of various instruments.

4. Risk Measurement Methods

4.1 VAR Approaches

We now describe the three major methods for computing VAR across large portfolios. The methods can be generally classified into **linear methods** and **full valuation methods**. Linear methods replace the positions by their linear exposures on risk factors, e.g. bonds by their dollar duration and options by their delta. Full valuation methods, in contrast, revalue all the instruments for the new values of the risk factors. Such methods are more complex and take longer to run but are generally more accurate.

The first method is called **delta-normal**, or **variance-covariance**. This involves, first, a linear mapping of the positions onto the risk factors, resulting in a vector of dollar exposures *x*. So, this is a linear valuation method. Next, the risk manager computes the covariance matrix of the risk factors Σ, typically from historical data. This can be constructed to place more weight on more recent data, as in the EWMA approach. The variance of the portfolio is then computed from $\sigma_p^2 = x'\Sigma\ x$. Assuming for instance a normal distribution gives

$$VAR = \alpha\sigma_p = \alpha\sqrt{x'\Sigma\ x} \tag{25}$$

This method is very simple and quick to implement. Unfortunately, it is inappropriate if the portfolio has non-linear instruments such as options, or if its distribution is strongly non-normal. If it is symmetric, however, a simple solution is to use the multiplier α from a distribution with fatter tails.

The second method is called **historical-simulation**. This is a full valuation method that simulates movements in the risk factors from the recent history. The current portfolio value is P_t, which is a function of the *N* current risk factors at time *t*, $P_t = P[f_{1,t}, f_{2,t}, \ldots, f_{N,t}]$. We sample the changes in factor movements from the historical distribution, without replacement. The first change *k*=1 comes from yesterday's movements *j*=*t*-1, the second from the day before and so on

$$\Delta f^k = \{\Delta f_{1,j}, \Delta f_{2,j}, \ldots, \Delta f_{N,j}\} \tag{26}$$

Next, we construct hypothetical factor values, starting from the current ones. For factor *i*, this is $f_i^k = f_{i,t} + \Delta f_{i,j}$. These are used to reprice the portfolio $P^k = P[f_1^k, f_2^k, \ldots, f_N^k]$. We can then sort the portfolio values to build the distribution of returns. VAR is then computed from the sample quantile. This method is widely used because it can handle options and historical returns distributions. On the other hand, it relies on a typically short moving window (1 to 4 years) to infer the factor distribution. If this **backward-looking window** does not contain some major risks or covers an unusually quiet period, the method will understate risk.

The third method is called **Monte Carlo simulation**. This is very similar to the historical simulation period except that factor movements are sampled from a pre-specified distribution

$$\Delta f^k \approx g(\Delta f; \theta) \tag{27}$$

where *g* is the joint distribution and θ the parameters. We could run millions of simulated scenarios *k*, each case revaluing the entire portfolio. VAR is then computed from the distribution of changes in portfolio values. This method is very flexible because it can accommodate many types of stochastic processes. On the other hand, it will take more computational time and is less intuitive than other methods. Mistakes in the specification of the model will not be as easy to identify. Thus, this approach is more powerful but is subject to **model risk**.

4.2 Risk Decomposition

The goal of risk measurement systems should be to provide much more than a single summary measure of risk. They should also help the portfolio manager understand the sources of risk and drill down to the level of subportfolios and even individual positions. **Marginal risk** provides such information, representing the change in risk due to a small increase in one of the allocations. For simplicity, we can focus on risk measures that are based on the standard deviation because these lead to analytical expressions. Define x_i as the size of the dollar position in asset or risk factor *i*. Using VAR = $\alpha\sigma_P W$ as the risk measure, the marginal risk of position *i* in portfolio *P*, MRISK, is

$$\text{MRISK}_i = \frac{\partial \text{VAR}}{\partial x_i} = \frac{\partial(\alpha\sigma_P W)}{\partial x_i} = \alpha \frac{Cov(R_i, R_P)}{\sigma_P} = \alpha\ \beta_{i,P}\sigma_P = \alpha\ \rho_{i,P}\sigma_i \qquad (28)$$

Using VAR = $\alpha\sigma_P W$, the MRISK of an allocation is given by $(\text{VAR}/W)\beta_{i,p}$. This means that the change in the VAR of a portfolio resulting from a small change in the size of a position is proportional to the beta of the position with respect to the portfolio.

MRISK is a unitless measure because it is constructed as the ratio of a dollar VAR to a dollar change in the position. Here, β is defined from a regression of risk factor *i* on the portfolio. A large value for β indicates that a small addition to this position will have a relatively large effect on the portfolio risk. Hence, positions with large betas should be cut first because they will lead to the greatest reduction in risk. Alternatively, the positions can be kept in the portfolio if they have comparatively high expected returns. Whichever is the choice, the portfolio manager should be fully aware of the risk implications of the positions.

This tool can be expanded to measure the contribution to the portfolio risk, CRISK, which is obtained by multiplying the marginal risk for position *i* by its weight in the portfolio

$$\text{CRISK}_i = x_i(\alpha\beta_{i,P}\sigma_P) = x_i \times \text{MRISK}_i \qquad (29)$$

Component VAR is measured in dollars, like VAR. Given the definition of MRISK from above, we can see that the CRISK of a position is $VAR \times w_i \times \beta_{i,p}$, where w_i is the weight of position i in the portfolio. We can write x_i in terms of w_i times the dollar value *W* of the portfolio: $x_i = w_i W$. Because the beta of a portfolio with itself is one, the weighted sum of $w_i\ \beta_{i,P}$ across the *N* risk factors is guaranteed to be one. Hence, this proves that the sum of the risk contributions adds up exactly to the total portfolio risk, RISK:

$$\text{RISK} = \alpha\sigma_P W = \alpha(\textstyle\sum_{i=1}^{N} w_i\beta_{i,P})\sigma_P W = \sum_{i=1}^{N} x_i(\alpha\beta_{i,P}\sigma_P) = \sum_{i=1}^{N} \text{CRISK}_i \qquad (30)$$

Therefore, we have shown how to decompose RISK into an additive and exhaustive decomposition.

Component VAR provides an additive decomposition of the portfolio VAR. This decomposition is not obvious because it depends on the weight of each risk factor in the portfolio, its volatility, and its correlation to the entire portfolio. Positions that hedge the portfolio risk will have negative component VAR. Positions can be ranked in order of decreasing importance of component VAR. Those at the top, generally above 5 percent of the total, are called **hot spots**. They should be closely examined by the portfolio manager because they contribute most to the risk of the portfolio. The portfolio manager should make sure that these are not unintended bets, but rather that they are justified by views.

As an example, consider our previous portfolio that was short \$4,000 million yen, to which is added a long position of \$1,000 million in euros. The two currencies have a slightly positive correlation of 0.28. Table 4 displays the risk decomposition.

Recall that the stand-alone position in the yen had a daily VAR of $43.7 million at the 95% level of confidence. The combined portfolio now has a total VAR of $41.9 million, which is lower due to diversification effects. For the yen position, the marginal VAR is the change in portfolio VAR after adding $1 million to the position. If so, VAR changes from $41.9392 to $41.9286, which is a change of –0.0106. Therefore the negative marginal VAR entry for the yen indicates that the adding to the position, or bringing it towards zero, should reduce risk.

Table 4. Risk Decomposition of Currency Portfolio

	Market Value x_i	Volatility σ_i	Risk $\alpha\sigma_i W$	Marginal Risk MRISK_i	Component Risk $x_i\,\text{MRISK}_i$
$/Yen	-$4,000	0.66%	$43.7	-0.0106	$42.4
$/EUR	$1,000	0.63%	$10.4	-0.0005	-$0.5
Total	-$3,000		$41.9		$41.9

Next, multiplying this by the position of –$4,000 gives a component VAR of $42.4 for the yen. The component VAR for the euro is negative, reflecting the diversification benefit of adding this second currency to the portfolio.

In this case, the risk decomposition clearly shows that the risk of the total portfolio is driven by the position in the yen. The portfolio manager should have a strong view on the yen to justify the risk taken. In contrast, the position in the euro can be justified simply on risk reduction grounds.

More generally, this analysis can be done in reverse. **Risk budgeting** is the process by which an investor selects a total risk budget for the fund, which is then parceled down to various investments and positions. In this case, the focus is on the risk allocation instead of the usual market value allocation.

4.3 Stress Tests

As previously mentioned, the statistical distribution of risk factors is typically estimated over a short historical window. This may miss major movements in risk factors that occur infrequently. As a result, VAR measures must be complemented by stress tests. Risk managers typically assess extreme scenarios such as the stock market crash of 1987, currency devaluations, the credit crisis that started in 2007, and so on.

In fact, the Federal Reserve Bank applied a stress test to large U.S. banks to ascertain whether they could absorb losses in an adverse economic environment during 2009 and 2010. Scenarios that form the basis of stress tests can be taken from **historical** episodes. Alternatively, **prospective scenarios** are built from scratch, specifically tailored to the fund’s portfolio. The Basel Committee (2009) describes how to construct stress scenarios. VAR systems can easily accommodate new scenarios, which are handled just like any other period in the historical simulation window.

Scenario analysis is also routinely used to set margin requirements by prime brokers and clearing counterparties, often in combination with VAR measures. As an example, consider a portfolio with many short and long option positions on the same underlying asset. In this case, notional amounts are rather meaningless. Some of the positions could be fully or partially offsetting each other. The risk could be much greater, or less than the net amount initially invested. The counterparty would make sure that the margin requirement is sufficient by building a battery of scenarios with a range of movements in the asset price and its implied volatility. The entire portfolio is repriced in each scenario. The margin is then set as the worst loss across all scenarios. The advantage of scenarios is that they can help to uncover situations that are plausible, yet have no recent historical precedent. Thus stress tests are absolutely necessary complements to statistical risk measures such as VAR.

5. Illiquidity

So far, we have assumed that the balance sheet of the fund was rather liquid. This is generally the case for some categories of alternative investments such as global macro funds, commodity trading advisors (CTAs), and long/short equity funds. These funds invest in major currencies, large stocks, Treasury bills and bonds, which are very liquid. Instruments traded on exchanges are generally more liquid than **Over-The-Counter** (OTC) instruments. Less liquid are funds that invest in corporate debt. At the lower end of the range are real estate funds, private equity and venture capital funds, where transactions cannot be conducted for years.

5.1. Illiquidity and Risk Measures

Risk measures are negatively affected by asset illiquidity, which is the risk of losses due to the market impact of liquidating the positions. Illiquid assets trade infrequently. They have wide bid-ask spreads and large price impact. The **price impact function** describes how far down the price would have to move to sell a specific position.

Instrument liquidity risk creates a major problem for the measurement of risk. After all, risk measures should represent changes in market prices. If prices do not change, traditional risk measures cannot be accurate. Worse, they will be systematically biased downwards.

Consider for example a private equity fund that invests in distressed debt, i.e., debt issued by companies in financial distress or in bankruptcy. This debt trades infrequently, perhaps once a month. Typically, these funds report their net asset value at the end of each month. If the bonds are not liquid, it is unlikely that all bonds will have market-clearing prices on the last day of the month. Instead, the valuation could be based on a trade in the middle of the month. This is why the end-of-month price is called **stale**. Unfortunately, this distorts several risk measures.[12]

The first effect is that the reported monthly volatility is biased downward. This is because prices are based on trades during the month, which is similar to an averaging process. Movements in monthly averages are less volatile than movements based on end-of-period values. As an example, a moving average of a price with a window of 20 days will be smoother than the most recent price.

The second effect is that monthly changes will display positive autocorrelation. A movement in one direction will be only partially captured during one month if prices are stale. The following month, part of the same movement will show up in the return. This autocorrelation can be measured using the regression

$$R_t = \alpha + \rho R_{t-1} + \varepsilon \tag{31}$$

where ρ is the autocorrelation coefficient. It is called first-order because it relates returns to those lagged by one period. Positive values above 0.1 indicate potential illiquidity problems.

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[12] See also Getmansky et al. (2004).

This positive autocorrelation substantially increases the volatility over longer horizons. Consider the example in Equation (17) where we extrapolated the one-period volatility to two periods. Initially, we assumed that movements were uncorrelated across periods, which led to the square root of time rule, an adjustment of $\sqrt{2}$ =1.41. Now assume a non-zero first-order autocorrelation coefficient ρ. The multiple-period variance is now

$$\sigma_{12}^2 = \sigma_1^2 + \sigma_2^2 + 2\rho\sigma_1\sigma_2 = \sigma_1^2 + \sigma_1^2 + 2\rho\sigma_1^2 = \sigma_1^2 2(1+\rho) \qquad (32)$$

With an autocorrelation of ρ = 0.5, the adjustment factor to the volatility changes from $\sqrt{2} = 1.41$ to $\sqrt{2(1+\rho)} = 1.73$. Thus the risk should be higher by (1.73-1.41)/1.41 = 22%. In general, the variance over N periods can be written as

$$V(\sum_{i=1}^{N} R_i) = \sigma_1^2[N + 2(N-1)\rho + 2(N-2)\rho^2 + ... + 2(1)\rho^{N-1}] \qquad (33)$$

As a result, the widespread method of annualizing monthly data by multiplying by the square root of 12 understates the annual risk. This can be adjusted, however, using Equation (33) instead.

Alternatively, we can construct an adjusted series

$$R_t^* = \frac{1}{1-\rho} R_t - \frac{\rho}{1-\rho} R_{t-1} \qquad (34)$$

When ρ = 0, this collapses to the usual return $R^*_t = R_t$. A positive value for ρ increases the volatility of the adjusted series R^*_t.[13] This adjustment method was originally developed to deal with the observed smoothing of real estate prices. Because of the long time needed to close a real estate transaction and high transaction costs, prices do not immediately adjust to new information.

A third, related effect is that measures of systematic risk will be systematically biased downward. Consider an asset with a monthly return of R_t. If the market I_t goes up during a month, only a fraction of this increase will be reflected in the NAV, leading to a beta measure that is too low. This can be corrected using lags

$$R_t = \alpha + \beta_0 I_t + \beta_{-1} I_{t-1} + \beta_{-2} I_{t-2} + \varepsilon \qquad (35)$$

The corrected beta is then the sum of the contemporaneous beta plus betas on lagged values of the index. This beta, called **Dimson beta**, is defined as $\hat{\beta} = \beta_0 + \beta_{-1} + \beta_{-2}$. Here we arbitrarily included two lagged values of the index. In practice, lags would be added up to the point where their coefficient is no longer significant.

More generally, correlations of illiquid assets with other asset classes are biased downward. This is a serious issue when "low correlations" are used as a major argument for investing in new asset classes.

Equations (34) and (35) provide an adjustment to risk measures for short-term returns, typically monthly. Another approach is to extend the return interval, e.g., to take quarterly or even annual steps instead of monthly steps. This second method is simpler. Unfortunately, it leads to less precise risk estimates because the number of independent data points shrinks quickly. For example, ten years of monthly data yield 120 data points for the monthly volatility and beta estimates. Using annual returns relies on 10 data points only.

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[13] Peterson and Grier (2006) explain how to adjust returns series that are artificially smooth for the purpose of computing covariance matrices, which are essential inputs into asset allocation.

Even with these statistical adjustments, historical data have limitations. For private equity (PE) funds, valuations are based on unrealized as well as realized investments, which introduces noise and potential biases due to subjective accounting treatment. Even here, position-based information can be useful to improve risk measures. For private equity, positions in non-traded stocks could be replaced by positions in traded stocks in equivalent industries, countries, and of like size. This mapping process would certainly create better risk measures than those based on investments carried at cost. Ljunqgvist and Richardson (2003), for example, estimate the systematic risk of PE funds by identifying the companies held in each fund and assigning them the beta of publicly-traded firms in the same industry. They report an average beta of 1.1. Hence, PE funds that are more leveraged than typical public equities can have a very high beta. They also find that PE funds tend to be concentrated in one or two industries, which must create higher risk.

Illiquidity can have a major effect on the risk-adjusted performance of alternative investments. For instance, performance is often evaluated with the **Sharpe ratio** (*SR*), which is the ratio of the average return on the portfolio $\overline{R}_P$ in excess of the risk-free rate over the volatility

$$SR_p = \frac{\overline{R}_P - R_F}{\sigma_P} \tag{36}$$

Table 5 compares the total returns on indices representing (1) publicly traded US stocks (S&P 500 Index), (2) hedge funds (CSFB Global Index), and (3) private equity (PE) funds (Cambridge Associates Index). The table displays the annualized performance estimated from quarterly data measured over the period from 1994 to 2005. For example, the usual risk measures panel shows that the volatility of PE is 11.7%, which appears lower than the 16.8% risk of US stocks. The beta of PE is 0.54 only. These numbers are misleading, however, because the autocorrelation of the PE index is very high, at 0.45.

Table 5. Comparison of Performance of U.S. Stocks, Hedge Funds, and Private Equity over 1994-2008 (from Quarterly Data, Annualized)

Asset Class	Average	Usual Risk Measures				Adjusted Risk Measures		
		Std.Dev.	Beta	Sharpe Ratio	Autocor-relation	Std.Dev.	Beta	Sharpe Ratio
US Stocks	7.7%	16.8%	1.00	0.24	0.09	16.8%	1.00	0.24
Hedge Funds	8.9%	8.9%	0.35	0.57	0.23	10.6%	0.35	0.48
Private Equity	14.0%	11.7%	0.54	0.88	0.45	16.2%	0.86	0.64

Source: Author's computations. The "usual risk measures" transform the quarterly standard deviation to an annualized measure by multiplying by the square root of four. The "adjusted risk measures" take autocorrelation into account and adjust the standard deviation, beta, and Sharpe ratio accordingly.

The right panel reports adjusted risk measures using Equations (33) and (35), the latter with three lags. The volatility and beta of the PE index are now markedly higher, at 16.2% and 0.86, respectively. Taking annual steps produces similar results, with estimates of 17.4% and 1.07. As a result, the Sharpe ratio, which appeared several times higher than that of US stocks, drops from 0.88 to 0.64, a considerable difference.

For the hedge fund index, the corrections are minor. The autocorrelation is small, leading to slightly higher volatility. There are no significant lags on the market. The Sharpe ratio drops from 0.57 to 0.48, which is a lesser change. The risk-adjusted performance is still twice that for US stocks.

Thus it is important to correct for illiquidity effects when evaluating risk-adjusted performance. Conroy and Harris (2007) reach even stronger conclusions. Based on a number of other indices over the period 1989 to 2005, they show that the volatility and beta of private equity are higher than that of US stocks. As a result, they argue that when correctly adjusted for risk, the performance of private equity has been hardly better than that of US equities.

5.2. Forced Liquidation Risk

Illiquidity causes another type of risk, which cannot be as easily measured as market risk. Funds that are leveraged may face funding requirements that could force them to sell assets in order to raise cash. Thus, **funding liquidity risk**, which arises when the firm cannot meet cash flow or collateral needs, can cause **asset liquidity risk**, which is the risk of losses due to the price impact of large asset sales. Liquidity risk, however, is complex and difficult to reduce to simple quantitative rules.

Commercial banks are naturally exposed to this type of risk. On the liability side, they raise deposits, a form of short-term debt, that are used to invest in long-term assets, such as loans. Even if the bank is solvent, i.e., the value of assets exceed that of liabilities, it might run into difficulties if depositors demand their money back all at once. In other words, this describes a “bank run”.

Hedge funds are also exposed to liquidation risk, especially when they have high leverage. Table 6 links sources of liquidation risk to a hedge fund balance sheet. Asset liquidity risk arises on the asset side and is a function of the size of the positions as well as of the price impact of a trade. On the liabilities side, funding risk arises when the hedge fund cannot rollover funding from its broker, or when losses in marked-to-market positions or increases in haircuts lead to margin cash requirements. This is often a major source of risk for hedge funds because the failure to meet margin calls can lead the lender to seize the collateral, forcing liquidation of the fund. In these situations, the portfolio manager loses control of the investment strategy, which can lead to a **blowup**. Finally, funding risk also arises when the fund faces investor redemptions.

Table 6. Balance Sheet and Sources of Liquidation Risk for a Hedge Fund

Assets	**Liabilities**
Size of positions	Funding
Price impact	Mark-to-market, haircuts
	Equity
	Investor redemptions

Alternatives managers typically try to manage their liquidation risk by matching the horizon of their assets and liabilities. Funds that invest in highly liquid assets, such as CTAs that deal in exchange-traded futures, can allow daily investor redemptions. On the other hand, funds that invest in illiquid securities, such as distressed debt, should impose long **lockup periods**, meaning that investors cannot redeem their investment for an extended, set period of time. For hedge funds, lockup periods average three months but they can extend up to five years. When redemptions are allowed, a minimum **notice period** can be required. Funds also often have **gates**, which limit the amount of withdrawals each period to a fraction of the equity investment. In extreme cases, funds generally have the ability to impose an outright **suspension** of redemptions.

For private equity funds, whether illiquidity is a problem depends on the capital structure. Some PE categories have no leverage. An example is **venture capital** funds, which involves equity investments in start-up ventures. Because there is no debt, the asset side is matched with the liability side, consisting of investor equity that may not be redeemed for a long period. Other categories have leverage. The best example consists of **leveraged buyouts** (LBOs), where public firms go private by repurchasing all outstanding shares. The acquisition is financed by a large proportion of debt, typically from 60% to 80% of the transaction value. This can include **senior debt** and **subordinated debt**, also called **mezzanine debt**. Here the risk is that of not being able to roll over the debt. Often, however, a large fraction of debt consists of short-term bank bridge loans, which may have to be repaid after two years only. This could cause liquidity problems. During the market turmoil that started in 2007, bank refinancing indeed became very difficult. In response, PE firms issued **capital calls** to their investors.[14] Such capital calls help PE firms manage their liquidity risk.[15]

6. Limitations of Conventional Risk Measures

6.1. General Limitations

A good risk manager should be keenly aware of the limitations of conventional risk measures. First, while statistical risk measures such as VAR are designed to give a sense of the potential extent of losses, they certainly do not describe the absolute worst loss. The risk manager can increase the confidence level so as to experience fewer exceptions but this is going to create other problems. Due to the paucity of data in the tails, the VAR measures are increasingly unreliable at higher confidence levels, even when distributions are stationary.

Second, modern risk measures are based on current positions that are assumed fixed over the time horizon. In practice, dynamic trading could increase or decrease risk. Such changes can be identified by backtesting both actual returns and **hypothetical returns.** The latter recreate the holding period return assuming a frozen portfolio. If the backtest fails for actual returns but not hypothetical returns, the risk manager can conclude that the model is well calibrated but that actual trading increases the risk profile.

Third, as previously mentioned, all risk systems involve simplifications, obtained by mapping the positions on the selected risk factors. These simplifications could create “holes” in the risk systems. Many hedge funds, for example, take positions in corporate bonds that are hedged by purchasing credit default swaps (CDS). Normally, losses on the bonds should be offset by gains on the CDSs. If the risk system maps both bonds and CDSs on the same curve, the net exposure is zero, which makes it look as if there is no risk. During 2008, however, the basis between bond and CDS spreads widened sharply, causing mark-to-market losses for many

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[14] A “capital call” occurs when a PE manager, usually the general partner, requests than an investor in the fund (a limited partner) provide additional capital. When entering a new PE investment, a limited partner typically injects initial funding and also agrees to provide additional capital over time, up to a maximum amount.

[15] The other side of the coin, however, is that investors may be forced to invest additional money precisely at the same time as turmoil in financial markets is creating losses on the rest of their portfolio. Siegel (2008) argues that these liquidity considerations are important when evaluating allocations across asset classes.

funds. These losses were not anticipated by most risk systems. Thus, the design of risk management systems depends on the trading strategy and requires experienced risk managers.

More generally, **model risk** can occur at various stages of the risk management process. Figure 6 shows that errors can arise when trades and market data are entered into the system, when risk factors are statistically modeled, during the mapping process, and even during implementation.

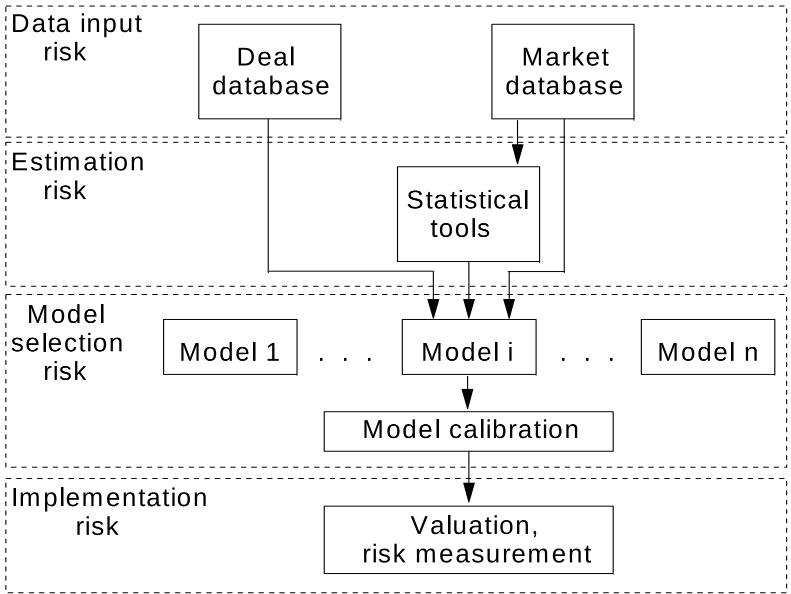

Fig. 6. Model Risk

Finally, most statistical risk measures assume that the recent past is a good representation of the future. This may not be the case, however, if the recent past has been unusually quiet or if it contains none of the events that are likely to develop in the future. As the Counterparty Risk Management Policy Group (2008) put it, "Risk monitoring and risk management cannot be left to quantitative risk metrics, which by nature are backward looking." This is why stress tests are required as well. This is particularly an issue in a period of rising volatility. Models based on simple moving averages respond slowly to these changes and systematically underestimate future risk. Models such as the EWMA that place more weight on more recent data will respond more quickly to rising volatility.

Even so, some risks are totally outside the scope of most scenarios. In 2008, many risk models largely failed due to "unknown unknowns." This includes **regulatory risks** such as the sudden restrictions on short-sales, which wreaked havoc on hedging strategies, or structural changes such as the conversion of investment banks to commercial banks, which accelerated the deleveraging of the financial industry. Similarly, it is difficult to account fully for **counterparty risk**. It is not enough to know your counterparty; you need to know your counterparty's counterparties too. In other words, these are network externalities. Understanding the full consequences of Lehman Brothers' failure would have required information on the entire topology of the financial network. Such contagion effects transform traditional risks into **systemic risk**, which can only be handled by the regulators or the government.

6.3. Things to Watch For

Risk managers should thoroughly understand the risk profile of the investment strategy. Some types of investments, such as small stocks or private equity, involve an upfront investment that can be returned several times if successful. This strategy is similar to a **long option** position, where the upfront payment is the maximum loss. As shown in Figure 7, this type of distribution has a long right tail, or positive skewness, which is a desirable feature. Long option positions can only lose the premium but can return many times this amount.

Such distributions can also be created by dynamic trading. For example, adding to a position after experiencing gains replicates the payoff from a long option position. This is typical of many **trend-following systems**. Similarly, **stop-loss rules** cut positions after losses are incurred. Traditional risk measures, however, assume that the portfolio is fixed and may miss this behavior. In such cases, traditional risk measures will overestimate risk, which is conservative.

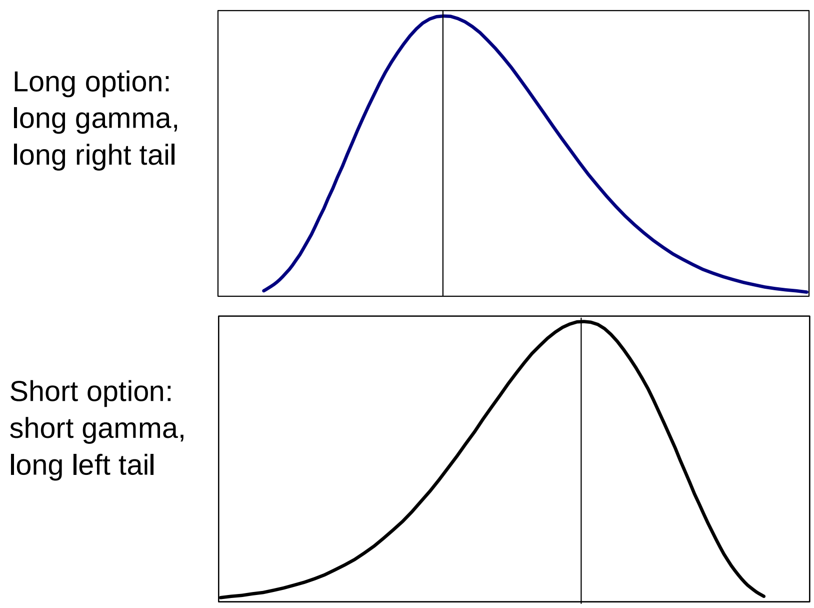

Fig. 7. Distribution of Investment Payoffs

Much more dangerous are situations where the portfolio manager takes positions in instruments with embedded **short option** positions, or when dynamic trading replicates a short option position. In these cases, the distribution has a long left tail, meaning that the investment is exposed to very large losses.

These positions are quite common, unfortunately. For instance, any investment in a credit-sensitive bond can be decomposed into a long position in a risk-free bond plus a short position in a credit default swap. The best that can happen is that all the coupons and principal will be paid back in time, in which case the actual return is basically the yield to maturity. The upside is limited in this case. On the other hand, a default can completely wipe out the investment.

The problem is that portfolio managers tend to like short positions in options because they deliver consistent outperformance—as long as the options are not exercised. A good example is that of Bernard Madoff, who reportedly lost $50 billion of investor money. He attracted so much money because his funds delivered good and steady but not high returns, which turned out to have been fabricated. This fraud is the largest Ponzi scheme in history.

A different example is that of UBS. During 2007, UBS suffered losses of $19 billion on super senior, triple-A rated, tranches of pools backed by mortgage debt, also called asset-backed securities (ABS). Investing in super senior tranches can be viewed as selling out-of-the-money put options. As long as the real estate market continued to go up, the default rate on subprime debt was relatively low and the super senior debt was safe, experiencing no price volatility. However, as the real estate market started to correct in 2007, the "put options" moved in-the-money, which led to large losses on the super senior debt.

Of course, none of these movements showed up in the previous historical data because of the sustained appreciation in the housing market up to 2007, but also because of the inherent nonlinearity in these securities. Instead of modeling how these structures depended on real estate prices, some banks simply chose to "map" the super senior debt on AAA-rated corporate bond curves. This gross simplification assumed that these tranches had no credit risk and totally ignored their nonlinearities. This is an example of a flawed mapping process.

As a result, UBS did not impose internal risk-based capital charges for units of the bank that invested in these asset-backed securities. Because these securities returned a wide spread over LIBOR and capital was only charged LIBOR internally, this was an arbitrage opportunity. Not unexpectedly, these securities found their way into the CDO warehousing book, into the trading book, into the liquid Treasury book, and into a hedge fund subsidiary. As reported later by UBS (2008), there was no monitoring of net or gross concentrations of positions in this asset class at the firm-wide level. By the start of 2007, the notional exposure had grown to approximately $100 billion. At the time, UBS had about $33 billion in tangible equity capital.

In conclusion, while the credit crisis that started in 2007 admittedly led to extreme and totally unexpected movements in risk factors, there were also notable failures in some risk models. As the Senior Supervisor Group (2008) report indicates, financial institutions that did poorly used outdated or inflexible assumptions in their risk models. These examples demonstrate the limitations of risk systems. Risk managers should be aware of potential weaknesses in conventional risk measures and continuously reassess their effectiveness.

7. Transparency

7.1. Problems with Non-Transparency

Managers of alternative investments are generally reluctant to reveal information about their positions. This lack of transparency has serious disadvantages for investors, however.

Disclosure allows **risk monitoring** of the fund, which is especially useful with active trading. This can help to avoid situations where the portfolio manager unexpectedly increases leverage or changes style. Closer monitoring of the fund can also decrease the probability of fraud and, more generally, blowups.[16]

Disclosure is also important for **risk aggregation**. The investor should know how the fund interacts with other assets in the portfolio. Whether the fund has a positive or negative correlation with the rest of the portfolio affects the total portfolio risk.

[16] Christory, et al. (2006) examine the characteristics of hedge funds that blowup. Over the 1994 to 2004, they report an average probability of default of 0.30% per annum. Most of the blowups observed are attributed to operational problems such as fraud, which can be minimized through a due diligence process and continuous monitoring.

In 2008, two blue-ribbon private-sector committees established by the President's Working Group (PWG) released separate sets of "best practices" for hedge fund investments. One report reflected the viewpoint of asset managers; the other report was written by investors. The two reports offer strikingly different perspectives on the need for disclosures and transparency. The investor committee (PWG, 2008a) states, "A key concern for investors is that hedge funds' lack of transparency may lead to unexpected risk exposures. ... Hedge fund managers typically cite commercial reasons for providing little transparency. There are sometimes legitimate competitive reasons for keeping information confidential, but often there are not." The term "transparency" is mentioned 16 times in this report, as opposed to "confidential," which is mentioned only once. In contrast, the term "transparency" is not even mentioned once in the asset manager report (PWG, 2008b), as opposed to "confidential" which is mentioned eight times.

Greater disclosure is resisted on the grounds that it would reveal proprietary information, leading to the possibility of a third-party trading against the fund. This threat, however, comes from the broker-dealer community, and generally not from investors. If this is an issue, confidentiality agreements should prevent leakages of sensitive information. AI managers generally prefer to release such information to investors with no trading operations, whether directly or through affiliates, who would not be able to profit from these data. Recipients of position-level information should have internal controls to prevent the dissemination and inappropriate use of this information.

Another argument which is sometimes advanced is the lack of investor sophistication. In other words, disclosing positions would give too much information to investors who might not be able to use it. This is a "paternalistic" view, however. Many investors do have the capabilities to use the information and should have the choice to do so.

7.2. Solutions for Transparency

These arguments can be addressed with a number of solutions, in particular for hedge funds. The first consists of external risk measurement services. These firms receive the individual positions of funds, after signing the proper non-disclosure agreements, and provide aggregate risk measures to investors. This solution partially solves the problems of risk aggregation and managers' widespread reluctance to disclose detailed information about their positions. On the other hand, risk service providers have little incentive to model risk as accurately as possible because they do not have a stake in the portfolio performance. They rarely perform backtesting, for example.

Another solution, which is still fairly rare, is to invest through a fund of funds that has position-level information. A fund of funds with no related trading operations is more likely to earn the trust of hedge fund managers. Also, large funds of funds should have the capabilities to process this information, as building risk systems is a complex undertaking that benefits from economies of scale. As a result, such funds of funds can perform the risk monitoring and measurement function for the investor. This position-level information can also be used to provide independent checks on the valuation of assets in the portfolio and to improve the portfolio construction process, thereby justifying the added fee for the fund of funds.

8. Conclusions

The alternative investments industry has thrived because of its good performance, which is explained by a combination of investment flexibility and strong financial incentives for fund managers. These features, however, should also cause concerns because they may lead fund managers to take on too much risk. Indeed, hedge funds failures, or blowups, seem to occur on a regular basis. Risk should be managed at the level of the fund, by the portfolio manager, and at the investor level, either directly or indirectly through risk aggregation services or funds of funds.

Relative to the traditional asset management industry, however, risk management is a special challenge for alternatives. Alternative products run the entire gamut of investment styles. At one end are CTAs with frenetic trading activity. At the other extreme is private equity, where investments are not traded, hard to value, and locked for years.

In each case, risk measures ideally should be based on position-level information. Returns-based risk measures have severe drawbacks. First, the length of the time series may not be long enough for meaningful risk analysis. Second, risk measures based on older data may no longer be relevant. This is especially an issue given the wide investment latitude given to some managers and how fast they can trade in and out of positions. Amaranth Advisors LLC, for instance, started as a convertible bond trading fund and then morphed into a predominantly highly leveraged natural gas trading fund. Such change would be very difficult to identify from returns data.

Illiquid assets pose different problems. Stale prices create biases in risk measures, causing volatility and systematic risk to be understated. This has implications for the role of these assets in portfolio allocation and for risk-adjusted performance measures.

Overall, this chapter has described several approaches to manage risk. Risk managers should use exposure measures, statistical risk measures, and stress tests. As we have seen, the design of an effective risk system requires a thorough understanding of the underlying trading strategies. It requires simplifications that recognize the trade-off between speed and accuracy. Overall, risk management for alternatives is still as much an art as a science. Using common sense is important when interpreting risk numbers.

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