Online Appendix to “Cost Analysis in Global Supply Chains”

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Appendix: Proofs

Proof of Proposition 1. Based on the equilibrium pricing decisions listed in §3.1, we can analyze the effect of supplier’s costs on those equilibrium prices. More specifically, we take the first order derivatives of each player’s price with respect to a supplier’s cost.

**Effect of \( C_1 \) on prices.** \[ \frac{\partial P_1}{\partial C_{1_i}} = \frac{7\tau^4-\tau^3-28\tau^2+2\tau+28}{12\tau^4-2\tau^3-50\tau^2+4\tau+48} \geq 0; \quad \frac{\partial P_2}{\partial C_{1_i}} = -\frac{\tau^2-2}{2(\tau^3+\tau-6)} \leq 0; \quad \frac{\partial P_3}{\partial C_{1_i}} = \frac{\tau^2-2}{2(\tau^3+\tau-6)} \geq 0 \]

if \( \tau \geq 0.282; \) \[ \frac{\partial P_4}{\partial C_{1_i}} = \frac{\tau^5-8\tau^4+4\tau^3+36\tau^2+4\tau-40}{4(\tau^3-4)(2\tau^2-\tau-4)(3\tau^4+\tau+6)} \geq 0; \quad \frac{\partial P_5}{\partial C_{1_i}} = \frac{5\tau^5+2\tau^4-24\tau^3-8\tau^2-28\tau+8}{2(\tau^3-4)(2\tau^2-\tau-4)(3\tau^4+\tau-6)} \geq 0 \]

**Effect of \( C_S \) on prices.** Since supplier 1 and supplier 2 are symmetric, the results for the effect of \( C_S \) can be derived accordingly.

Based on above results, it is straightforward to plot the effect of suppliers’ cost on prices against \( \tau \) in Figure 2. This completes the proof of Proposition 1.

Proof of Proposition 2. Based on the equilibrium pricing decisions listed in §3.1, we can calculate equilibrium profit of each player in the supply chain. Note that the equilibrium prices are linear in the cost parameters, which results with quadratic equilibrium profits in costs. This enables us to carry out a sensitivity analysis with respect to costs. More specifically, we take the first order derivatives of each player’s profit with respect to a supplier’s cost. This proposition considers the vertical cost effect, thus we analyze how a cost change in a supplier affects members that involve

A.1
in a vertical relationship with this supplier.

**Effect of \( C_{S_1} \) on supplier \( S_1 \), assembler \( A_1 \) and retailer \( R \).** We compare \( \frac{\partial \Pi_{S_1}}{\partial C_{S_1}}, \frac{\partial \Pi_{A_1}}{\partial C_{S_1}}, \) and \( \frac{\partial \Pi_{R}}{\partial C_{S_1}} \). In order to guarantee positive sales of the two final products, it is straightforward to show that \( Q_{R_1} \geq 0 \) when \( C_{S_1} \leq k''_{C_{S_1}} = \frac{-2(2\tau^4 + 3\tau^2 - 9\tau + 2\tau + 8)(C_{S_1} + 1) + C_{S_2}(\tau^4 - 3\tau^3 - 4\tau^2 + 6\tau + 4)}{5\tau^4 - \tau^2 - 22\tau^2 + 2\tau + 20} \), and \( Q_{R_2} \geq 0 \) when \( C_{S_1} \geq k''_{C_{S_1}} = \frac{2(2\tau^4 + 3\tau^2 - 9\tau + 2\tau + 8)(C_{S_1} - 1) + C_{S_2}(5\tau^4 - 3\tau^3 - 22\tau^2 + 2\tau + 20)}{(7\tau - 2)(\tau^2 - 3\tau - 2)} \), where \( k''_{C_{S_1}} \leq k'_{C_{S_1}} \).

One can easily verify that \( \frac{\partial \Pi_{S_1}}{\partial C_{S_1}} \leq 0 \) and \( \frac{\partial \Pi_{A_1}}{\partial C_{S_1}} \leq 0 \). However, \( \frac{\partial \Pi_{R}}{\partial C_{S_1}} \geq 0 \) when \( C_{S_1} \geq k''_{C_{S_1}} \). The whole comparison process can be divided into the following 3 parts and we only focus on the conditions when \( \frac{\partial \Pi_{R}}{\partial C_{S_1}} \leq 0 \):

**Part (1):** \( \frac{\partial \Pi_{S_1}}{\partial C_{S_1}} \geq 0 \) for all \( C_{S_1} \leq k'_{C_{S_1}} \). Thus \( \frac{|\partial \Pi_{S_1}|}{|\partial C_{S_1}|} \geq \frac{|\partial \Pi_{A_1}|}{|\partial C_{S_1}|} \).

**Part (2):** \( \frac{\partial \Pi_{S_1}}{\partial C_{S_1}} \geq 0 \) if \( C_{S_1} \leq k''_{C_{S_1}} \). \( \frac{\partial \Pi_{S_1}}{\partial C_{S_1}} \geq 0 \) if \( C_{S_1} \geq k_{C_{S_1}} \). Note that \( k''_{C_{S_1}} \geq k'_{C_{S_1}} \) only if \( \tau \geq 0.28 \). Therefore, when \( \tau \leq 0.28 \), \( \frac{\partial \Pi_{R}}{\partial C_{S_1}} \geq 0 \) if \( C_{S_1} \leq C_{S_1} \leq k_{C_{S_1}} \); when \( \tau \geq 0.28 \), \( \frac{\partial \Pi_{R}}{\partial C_{S_1}} \geq 0 \) for any given \( C_{S_1} \).

Combining the analysis in parts (1), (2) and (3), we are able to derive the results presented in Proposition 2 (1) and also summarized in Figure A.1.

**Effect of \( C_{S_2} \) on supplier \( S_2 \), assembler \( A_2 \) and retailer \( R \).** Since supplier 1 and supplier 2 are symmetric, the results for the effect of \( C_{S_2} \) can be derived accordingly.

**Effect of \( C_{S_\ell} \) on supplier \( S_{\ell} \), assemblers \( A_1 \) and \( A_2 \) and retailer \( R \).** In this part of
Therefore, the next step is to compare each pair of those values in the following 4 parts. In order to guarantee non-negative sales of the two final products, it is straightforward to show that $Q_{R1} \geq 0$ when $C_S \leq k''_{C_S} = \frac{C_S \left(t^4 - 3t^3 + 2t^2 - 2t - 20\right)}{2(t-1)(t+2)(2t^2 - t - 4)} + \frac{C_S \left(\tau^4 - 3\tau^3 + 4\tau^2 + 6\tau + 4\right)}{2(t-1)(t+2)(2t^2 - t - 4)} + \frac{(4\tau^4 + \tau^3 - 9\tau^2 - 2\tau + 8)}{(t-1)(t+2)(2t^2 - t - 4)}$ and $Q_{R2} \geq 0$ when $C_S \leq k''_{C_S} = \frac{C_S \left(\tau^4 - 3\tau^3 - 4\tau^2 + 6\tau + 4\right)}{2(t-1)(t+2)(2t^2 - t - 4)} + \frac{C_S \left(-5\tau^4 + \tau^3 + 2\tau^2 - 2t - 20\right)}{2(t-1)(t+2)(2t^2 - t - 4)} + \frac{(2\tau^4 + 5\tau^3 - 9\tau^2 + 2\tau + 8)}{(t-1)(t+2)(2t^2 - t - 4)}$. In addition, $k'_{C_S} - k''_{C_S} = (C_S - C_S) \left(3\tau^4 - 2\tau^3 - 13\tau^2 + 4\tau + 12\right)$, which is positive when $C_S < C_S$ and vice versa. Therefore, we restrict $C_S \leq k'_{C_S}$ when $C_S > C_S$, $C_S \leq k''_{C_S}$ when $C_S \leq C_S$, and $C_S \leq k'_{C_S} = k''_{C_S}$ when $C_S = C_S$.

One can also easily verify that $|\frac{\partial \Pi_1}{\partial C_S}| \leq 0$, $|\frac{\partial \Pi_2}{\partial C_S}| \leq 0$, $|\frac{\partial \Pi_3}{\partial C_S}| \leq 0$ and $|\frac{\partial \Pi_4}{\partial C_S}| \leq 0$. Based on this analysis, the next step is to compare each pair of those values in the following 4 parts.

**Part (1):** $|\frac{\partial \Pi_1}{\partial C_S} - \frac{\partial \Pi_2}{\partial C_S}| = \left|\left(C_S - C_S\right)\left(\tau^2 - 2\right)^2\right|$, which is positive when $C_S < C_S$.

Therefore, $|\frac{\partial \Pi_1}{\partial C_S}| > |\frac{\partial \Pi_2}{\partial C_S}|$ when $C_S < C_S$, $|\frac{\partial \Pi_1}{\partial C_S}| < |\frac{\partial \Pi_3}{\partial C_S}|$ when $C_S > C_S$, and $|\frac{\partial \Pi_1}{\partial C_S}| = |\frac{\partial \Pi_2}{\partial C_S}|$ when $C_S = C_S$.

**Part (2):** $|\frac{\partial \Pi_1}{\partial C_S} - \frac{\partial \Pi_2}{\partial C_S}| = \frac{\left(C_S + 2C_S + C_S - 2\right)(\tau - 3)(\tau^2 - 2)^2}{2(\tau - 2)(\tau - 1)(3\tau^2 - 2\tau - 6)} > 0$. Therefore, $|\frac{\partial \Pi_1}{\partial C_S}| > |\frac{\partial \Pi_2}{\partial C_S}|$.

**Part (3):** $|\frac{\partial \Pi_1}{\partial C_S} - \frac{\partial \Pi_2}{\partial C_S}| \geq 0$ if $C_S \leq k'_{C_S} = \frac{C_S \left(t^4 + 3t^3 - 2t^2 - 6t - 12\right) + C_S \left(-5t^4 + 5t^3 + 28t^2 - 14t - 36\right)}{2(t-3)(t+2)(2t^2 - t - 4)} + \frac{4t^4 - 6t^3 - 30t^2 + 20t + 48}{2(t-3)(t+2)(2t^2 - t - 4)}$. It is straightforward to show that when $C_S < C_S$, $|\frac{\partial \Pi_1}{\partial C_S}| < |\frac{\partial \Pi_2}{\partial C_S}|$ and thus $|\frac{\partial \Pi_1}{\partial C_S}| > |\frac{\partial \Pi_2}{\partial C_S}|$ when $C_S > C_S$, $k'_{C_S} > k'_{C_S}$ and thus $|\frac{\partial \Pi_1}{\partial C_S}| < |\frac{\partial \Pi_2}{\partial C_S}|$ when $C_S = C_S$.

A.3
When \( C \) have \( k \) that we restrict presented in Proposition 2 (2) and Figure A.2. symmetric to those results above by replacing \( A \) straightforward to show that thus we compare an individual supplier’s cost effect on the three suppliers in the upstream sup-

Proof of Proposition 3. This proposition focuses on the horizontal effect of suppliers’ costs, thus we compare an individual supplier’s cost effect on the three suppliers in the upstream supplier layer.

Effect of \( C_s \) on suppliers \( S_1, S_2 \) and \( S_c \). We compare \( \frac{\partial \Pi_1}{\partial C_s} \), \( \frac{\partial \Pi_2}{\partial C_s} \), and \( \frac{\partial \Pi_3}{\partial C_s} \). Recall that we restrict \( k''_{C_s} \leq C_s \leq k'_{C_s} \) to guarantee positive sales of the two final products. It is straight forward to show that \( \frac{\partial \Pi_1}{\partial C_s} \leq 0, \frac{\partial \Pi_2}{\partial C_s} \leq 0, \) and \( \frac{\partial \Pi_3}{\partial C_s} \geq 0 \). In addition, \( \frac{\partial \Pi_1}{\partial C_s} - \frac{\partial \Pi_2}{\partial C_s} \geq 0 \)
if \( C_S \geq k_{C_S}^{S_1} \), we have \( \frac{\partial \Pi_{S_1}}{\partial C_S} > \frac{\partial \Pi_{S_2}}{\partial C_S} \). Thus, when \( k_{C_S}^{S_1} \leq C_S \leq k_{C_S}^{S_1} \), we have \( \left| \frac{\partial \Pi_{S_1}}{\partial C_S} \right| \geq \left| \frac{\partial \Pi_{S_2}}{\partial C_S} \right| \), and when \( k_{C_S}^{S_1} \leq C_S \leq k_{C_S}^{S_1} \), we have \( \left| \frac{\partial \Pi_{S_1}}{\partial C_S} \right| \leq \left| \frac{\partial \Pi_{S_2}}{\partial C_S} \right| \). The results can also be summarized in Figure A.3.

\[
\frac{\partial \Pi_{S_1}}{\partial C_S} = \frac{\partial \Pi_{S_2}}{\partial C_S} > 0 \quad \text{if} \quad C_S \geq k_{C_S}^{S_1}
\]

Figure A.3: Horizontal Effect of the Dedicated Supplier \( S_1 \)'s Cost

**Effect of \( C_{S_s} \) on suppliers \( S_1, S_2 \) and \( S_c \).** Due to the fact that supplier \( S_1 \) and \( S_2 \) are symmetric, results for \( C_{S_s} \) can be derived accordingly by replacing \( C_{S_1} \) with \( C_{S_s} \) and \( \Pi_{S_1} \) with \( \Pi_{S_s} \) in the above analysis.

**Effect of \( C_{S_c} \) on suppliers \( S_1, S_2 \) and \( S_c \).** Next, we focus on the analysis of \( C_{S_c} \)'s horizontal effect. More specifically, we compare \( \frac{\partial \Pi_{S_1}}{\partial C_{S_c}}, \frac{\partial \Pi_{S_2}}{\partial C_{S_c}}, \) and \( \frac{\partial \Pi_{S_c}}{\partial C_{S_c}} \). Recall also that, in order to guarantee positive sales of the two final products, we restrict \( C_{S_c} \leq k_{C_{S_c}}^{S_1} \) when \( C_{S_1} \leq C_{S_2} \), and \( C_{S_c} \leq k_{C_{S_c}}^{S_1} \) when \( C_{S_1} \geq C_{S_2} \). One can easily verify that \( \frac{\partial \Pi_{S_1}}{\partial C_{S_c}} \leq 0, \frac{\partial \Pi_{S_2}}{\partial C_{S_c}} \leq 0, \) and \( \frac{\partial \Pi_{S_c}}{\partial C_{S_c}} \leq 0 \). The next step is to compare any two pairs of those values in the following two parts:

**Part (1):** \( \frac{\partial \Pi_{S_1}}{\partial C_{S_c}} - \frac{\partial \Pi_{S_2}}{\partial C_{S_c}} \geq 0 \) if \( C_{S_c} \geq k_{C_{S_c}}^{S_1} \), \( C_{S_1} \geq k_{C_{S_c}}^{S_1} \), \( k_{C_{S_c}}^{S_1} \geq k_{C_{S_c}}^{S_1} \) when \( C_{S_1} \leq C_{S_2} \), and \( k_{C_{S_c}}^{S_1} \geq k_{C_{S_c}}^{S_1} \) when \( C_{S_1} \geq C_{S_2} \). Thus \( \frac{\partial \Pi_{S_1}}{\partial C_{S_c}} \leq \frac{\partial \Pi_{S_2}}{\partial C_{S_c}} \). Similarly, we can also derive that \( \left| \frac{\partial \Pi_{S_1}}{\partial C_{S_c}} \right| \leq \left| \frac{\partial \Pi_{S_2}}{\partial C_{S_c}} \right| \).

**Part (2):** \( \frac{\partial \Pi_{S_1}}{\partial C_{S_c}} - \frac{\partial \Pi_{S_2}}{\partial C_{S_c}} = \frac{\partial \Pi_{S_1}}{\partial C_{S_c}} - \frac{\partial \Pi_{S_2}}{\partial C_{S_c}} = \frac{(C_{S_c} - C_{S_2})(t^2 - 2)}{|(2t^3 - 4t^2 + 4t - 8)|} \), which is positive when \( C_{S_1} \leq C_{S_2} \). Therefore, \( \left| \frac{\partial \Pi_{S_1}}{\partial C_{S_c}} \right| \geq \left| \frac{\partial \Pi_{S_2}}{\partial C_{S_c}} \right| \) when \( C_{S_1} \leq C_{S_2} \) and vice versa.

Combining the analysis of parts (1) and (2), we derive the result presented in Proposi-
Proof of Proposition 4. This proof shows that, for any individual member in the supply chain network, whether its profit would be affected most by either supplier \( S_1 \)’s cost change, or supplier \( S_2 \)’s cost change, or the common-component supplier \( S_c \)’s cost change.

Let us first focus on the upstream suppliers.

1. Supplier \( S_1 \): Recall that \( \frac{\partial P_{S_1}}{\partial C_{S_1}} \leq 0 \) and \( \frac{\partial P_{S_1}}{\partial C_{S_2}} \leq 0 \). One can easily verify that \( \frac{\partial P_{S_1}}{\partial C_{S_1}} - \frac{\partial P_{S_1}}{\partial C_{S_2}} \geq 0 \) if \( C_{S_1} \leq k'_{C_{S_1}} \), where \( k'_{C_{S_1}} \) is the higher bound to guarantee positive sales of \( Q_1 \). Thus \( |\frac{\partial P_{S_1}}{\partial C_{S_1}}| \geq |\frac{\partial P_{S_1}}{\partial C_{S_2}}| \). In addition, \( \frac{\partial P_{S_1}}{\partial C_{S_2}} \geq 0 \). This gives the result presented in Proposition 4(1) that supplier \( S_1 \) is negatively affected by its own cost increase in \( C_{S_1} \) more than an increase in \( C_{S_2} \) and positively affected by an increase in \( C_{S_2} \).

2. Supplier \( S_2 \): With similar approach, we can draw the same conclusion for supplier \( S_2 \) presented in Proposition 4(3).

3. Supplier \( S_c \). Recall that \( \frac{\partial P_{S_c}}{\partial C_{S_1}} \leq 0 \) and \( \frac{\partial P_{S_c}}{\partial C_{S_2}} \leq 0 \). It is straightforward to show that \( \frac{\partial P_{S_1}}{\partial C_{S_1}} - \frac{\partial P_{S_2}}{\partial C_{S_2}} = \frac{(2\tau + 2C_{S_2} + C_{S_1} - 2)}{2(\tau - 2)(\tau + 1)(3\tau + 2 - \tau)} \leq 0 \) and \( \frac{\partial P_{S_2}}{\partial C_{S_2}} = \frac{\partial P_{S_c}}{\partial C_{S_2}} \). Therefore supplier \( S_c \) is negatively affected by its own cost increase more than \( C_{S_1} \) and \( C_{S_2} \).

Next, we focus on the middle level assemblers. Recall that \( \frac{\partial P_{A_1}}{\partial C_{S_1}} \leq 0 \) and \( \frac{\partial P_{A_1}}{\partial C_{S_2}} \leq 0 \). One can easily verify that \( \frac{\partial P_{A_1}}{\partial C_{S_1}} - \frac{\partial P_{A_1}}{\partial C_{S_2}} \geq 0 \) if \( C_{S_1} \leq k'_{C_{S_1}} \), where \( k'_{C_{S_1}} \) is the higher bound to guarantee positive sales of \( Q_1 \). Thus \( |\frac{\partial P_{A_1}}{\partial C_{S_1}}| \geq |\frac{\partial P_{A_1}}{\partial C_{S_2}}| \). In addition, \( \frac{\partial P_{A_1}}{\partial C_{S_2}} \geq 0 \). This gives the result presented in Proposition 4(1) that assembler \( A_1 \) is negatively affected by an increase in \( C_{S_1} \) and positively affected by an increase in \( C_{S_2} \). With similar approach, we can draw the same conclusion for assembler \( A_2 \) presented in Proposition 4(3).

Lastly, we focus on the retailer’s profit. Recall from the proof of Proposition 2 that we always have \( \frac{\partial P_{R}}{\partial C_{S_c}} \leq 0 \). Thus, the retailer’s profit always decreases in \( C_{S_c} \).

Recall also that when \( \tau \leq 0.28 \), we have \( \frac{\partial P_{R}}{\partial C_{S_1}} \leq 0 \) if \( k'_{C_{S_1}} \leq C_{S_1} \leq k_{C_{S_1}} \) and \( \frac{\partial P_{R}}{\partial C_{S_2}} \geq 0 \) if \( k_{C_{S_1}} \leq C_{S_1} \leq k'_{C_{S_1}} \); when \( \tau > 0.28 \), we have \( \frac{\partial P_{R}}{\partial C_{S_1}} \leq 0 \) for any given \( k'_{C_{S_1}} \leq C_{S_1} \leq k_{C_{S_1}} \). Due to the symmetry position of supplier 1 and supplier 2, same results hold true by replacing \( C_{S_1} \) with \( C_{S_2} \).

Therefore, we draw the conclusion that the retailer’s profit decreases in \( C_{S_1} \) or \( C_{S_2} \) except when either \( C_{S_1} \) or \( C_{S_2} \) is high and \( \tau \) is low. It is natural for us to focus on the situation where retailer’s
profit decreases in all of the three costs. The situation requires either condition set (1) $\tau \leq 0.28$ and $k^\prime_{C_{S_1}} \leq C_{S_1} \leq k^R_{C_{S_1}}$, or condition set (2) $\tau \geq 0.28$ and $k^\prime_{C_{S_1}} \leq C_{S_1} \leq k^R_{C_{S_1}}$.

Consider the effect of an increase in $C_{S_1}$ and $C_{S_2}$ on the retailer’s profit. We have $\frac{\partial \Pi}{\partial C_{S_1}} = \frac{\partial \Pi}{\partial C_{S_2}}$, where $k^R_{C_{S_1}} \leq k^R_{C_{S_2}}$ for all $\tau$ and $k^R_{C_{S_1}} \leq k^R_{C_{S_2}}$ if $\tau \leq 0.28$ and vice versa. In addition, $k^R_{C_{S_1}}C_{S_2} \leq k^R_{C_{S_2}}$, and $|\frac{\partial \Pi}{\partial C_{S_1}}| \leq |\frac{\partial \Pi}{\partial C_{S_2}}|$. One can further verify that $k^R_{C_{S_1}}C_{S_2} = k^R_{C_{S_2}}$.

Consider the effect of an increase in $C_{S_2}$ and $C_{S_3}$ on the retailer’s profit. Similar results hold true by replacing $C_{S_1}$ with $C_{S_2}$ due to symmetry between suppliers $S_1$ and $S_2$. For simplification, we denote $k_1 = k^R_{C_{S_1}C_{S_2}} = k^R_{C_{S_1}}$ and $k_2 = k^R_{C_{S_1}C_{S_2}} = k^R_{C_{S_2}}$.

Next, consider the effect of an increase in $C_{S_1}$ and $C_{S_2}$ on the retailer’s profit. First of all, we have $\frac{\partial \Pi}{\partial C_{S_1}} - \frac{\partial \Pi}{\partial C_{S_2}} = \frac{\partial \Pi}{\partial C_{S_2}}(C_{S_1} - C_{S_2})(\tau^2 - 4\tau + 4)^2$, which is positive if $C_{S_1} \geq C_{S_2}$ and vice versa. Thus, $|\frac{\partial \Pi}{\partial C_{S_1}}| \geq |\frac{\partial \Pi}{\partial C_{S_2}}|$ when $C_{S_1} \leq C_{S_2}$ and $|\frac{\partial \Pi}{\partial C_{S_1}}| \leq |\frac{\partial \Pi}{\partial C_{S_2}}|$ when $C_{S_1} \geq C_{S_2}$.

Combining all the analysis above enables us to draw the following conclusions regarding the supplier’s cost effect on retailer’s profit, which can also be summarized in Figure A.4:

1. When $\tau \leq 0.282$ and $k_1 \leq C_{S_1} \leq k^R_{C_{S_1}}$, we have $\frac{\partial \Pi}{\partial C_{S_1}} \geq 0$ and $\frac{\partial \Pi}{\partial C_{S_2}} \geq |\frac{\partial \Pi}{\partial C_{S_1}}|$. 

2. When $\tau \leq 0.282$ and $k^\prime_{C_{S_1}} \leq C_{S_1} \leq k_2$, we have $\frac{\partial \Pi}{\partial C_{S_2}} \geq 0$ and $|\frac{\partial \Pi}{\partial C_{S_1}}| \geq |\frac{\partial \Pi}{\partial C_{S_2}}|$. 

3. When $\tau \leq 0.282$ and $k_2 \leq C_{S_1} \leq k_1$, the retailer’s profit decreases in all of the suppliers’ costs. Moreover, $|\frac{\partial \Pi}{\partial C_{S_1}}| \geq |\frac{\partial \Pi}{\partial C_{S_2}}| \geq |\frac{\partial \Pi}{\partial C_{S_3}}|$ if $C_{S_1} \leq C_{S_2}$, and $|\frac{\partial \Pi}{\partial C_{S_1}}| \geq |\frac{\partial \Pi}{\partial C_{S_2}}| \geq |\frac{\partial \Pi}{\partial C_{S_3}}|$ if $C_{S_1} \geq C_{S_2}$.

4. When $\tau \geq 0.282$ and $k^\prime_{C_{S_1}} \leq C_{S_1} \leq k^R_{C_{S_1}}$, the retailer’s profit also decreases in all of the suppliers’ costs. Moreover, $|\frac{\partial \Pi}{\partial C_{S_1}}| \geq |\frac{\partial \Pi}{\partial C_{S_2}}| \geq |\frac{\partial \Pi}{\partial C_{S_3}}|$ if $C_{S_1} \leq C_{S_2}$, and $|\frac{\partial \Pi}{\partial C_{S_1}}| \geq |\frac{\partial \Pi}{\partial C_{S_2}}| \geq |\frac{\partial \Pi}{\partial C_{S_3}}|$ if $C_{S_1} \geq C_{S_2}$.

Therefore, based on (3) and (4), we conclude that in the case when the retailer’s profit decreases in all of the three suppliers’ costs, an increase in $C_{S_2}$ has the strongest negative effect. In the case when the retailer benefits from an increase in $C_{S_1}$ (or $C_{S_2}$), an increase in $C_{S_3}$ has a less negative effect than an increase in $C_{S_2}$ (or $C_{S_1}$). This completes the proof.

**Proof of Proposition 5.** Proposition 5 aims to explore the effect of assemblers’ costs $C_{A_1}$ and $...
$C_{A_2}$. By using the same approach as the base model when we consider only the suppliers’ cost changes, we can derive the equilibrium quantities and each player's profit. Again, let us focus on the analysis of $C_{A_1}$. Results for the effect of $C_{A_2}$ can be derived accordingly.

In order to guarantee positive sales of the two final products, it is straightforward to show that $Q_{A_1} \geq 0$ when $C_{A_1} \leq k''_{C_{A_1}} = \frac{C_{A_2}(5t^4-\tau^3+22t^2+2\tau+20}{(2(t^3-3t^2+2t+20)}$, $Q_{A_2} \geq 0$ when $C_{A_1} \geq k''_{C_{A_1}} = \frac{C_{A_2}(5t^4-\tau^3+22t^2+2\tau+20)}{(2(t^3-3t^2+2t+20)}$, and $k''_{C_{A_1}} \leq k'_{C_{A_1}}$. We have $\frac{\partial P_{A_1}}{\partial C_{A_1}} \leq 0$, $\frac{\partial P_{A_1}}{\partial S_{A_1}} \leq 0$, and $\frac{\partial P_{A_1}}{\partial C_{A_1}} \geq 0$ when $C_{A_1} \geq k''_{C_{A_1}} = \frac{C_{A_2}(13t^7-13t^6-80t^5+76t^4+152t^3-140t^2-80t+80)}{5t^7-29t^6-26t^5+182t^4+40t^3-356t^2-16t+208}$ and $k''_{C_{A_1}} \leq C_{A_1} \leq k'_{C_{A_1}}$. Next we will compare $\frac{\partial P_{A_1}}{\partial C_{A_1}}, \frac{\partial P_{A_1}}{\partial S_{A_1}}$, and $\frac{\partial P_{A_1}}{\partial C_{A_1}}$. We only focus on the situation when $\frac{\partial P_{A_1}}{\partial C_{A_1}} \leq 0$. This leads to the analysis of the following 6 parts.

**Part (1):** $\frac{\partial P_{A_1}}{\partial C_{A_1}} - \frac{\partial P_{A_1}}{\partial S_{A_1}} \geq 0$ if $C_{A_1} \leq k^{S_{C_{A_1}}}S_{C_{A_1}}$, where $k^{S_{C_{A_1}}}S_{C_{A_1}} = \frac{C_{A_2}(13t^7-13t^6-80t^5+76t^4+152t^3-140t^2-80t+80)}{17t^6+77t^5-108t^4+32t^3+216t^2+28t-136}$ and $k^{S_{C_{A_1}}}S_{C_{A_1}} \leq C_{A_1} \leq k'_{C_{A_1}}$. Therefore, $|\frac{\partial P_{A_1}}{\partial C_{A_1}}| \leq |\frac{\partial P_{A_1}}{\partial S_{A_1}}|$ if $k''_{C_{A_1}} \leq C_{A_1} \leq k'_{C_{A_1}}$, and $|\frac{\partial P_{A_1}}{\partial C_{A_1}}| \geq |\frac{\partial P_{A_1}}{\partial S_{A_1}}|$ if $k''_{C_{A_1}} \leq C_{A_1} \leq k'_{C_{A_1}}$. Thus, $\frac{\partial P_{A_1}}{\partial C_{A_1}} \leq \frac{\partial P_{A_1}}{\partial S_{A_1}}$.

**Part (2):** $\frac{\partial P_{A_1}}{\partial C_{A_1}} - \frac{\partial P_{A_1}}{\partial S_{A_1}} \geq 0$ if $C_{A_1} \leq k'_{C_{A_1}}$, and $|\frac{\partial P_{A_1}}{\partial C_{A_1}}| \geq |\frac{\partial P_{A_1}}{\partial S_{A_1}}|$ if $k''_{C_{A_1}} \leq C_{A_1} \leq k'_{C_{A_1}}$. Thus, $\frac{\partial P_{A_1}}{\partial C_{A_1}} \leq \frac{\partial P_{A_1}}{\partial S_{A_1}}$. 

Figure A.4: Effect of the Dedicated Supplier $S_1$’s Cost on the Retailer’s Profit
Proof of Proposition 6. This proof focuses on the analysis regarding the effect of the retailer’s cost
C.R. By using a similar approach as used in the base model when we consider only the suppliers’
costs, we can derive the equilibrium quantities and each player’s profit. Given that supplier 1 and
supplier 2 are symmetric, and assembler 1 and assembler 2 are also symmetric, Π_S1 = Π_S2 and
Π_A1 = Π_A2. Thus we only need to compare \( \frac{\partial \Pi}{\partial C_R} \leq 0 \), \( \frac{\partial \Pi_{A_1}}{\partial C_R} \leq 0 \), \( \frac{\partial \Pi_{S_1}}{\partial C_R} \leq 0 \) and \( \frac{\partial \Pi_{S_2}}{\partial C_R} \leq 0 \).

It is straight forward to show that \( \frac{\partial \Pi_R}{\partial C_R} - \frac{\partial \Pi_{A_1}}{\partial C_R} = \frac{\tau (\tau^2 - 2)^2 (C_R - 1)}{(\tau - 2)^2 (\tau + 1)(3\tau^2 + \tau - 6)^2} \geq 0 \) if \( \tau \geq 0.54 \) and vice versa; \( \frac{\partial \Pi_R}{\partial C_R} - \frac{\partial \Pi_{S_1}}{\partial C_R} = \frac{(2\tau - 3)(\tau^2 - 2)^2 (C_R - 1)}{(\tau - 2)^2 (\tau + 1)(3\tau^2 + \tau - 6)^2} \leq 0 \); \( \frac{\partial \Pi_{A_1}}{\partial C_R} - \frac{\partial \Pi_{S_1}}{\partial C_R} = \frac{(\tau - 3)(\tau^2 - 2)^2 (C_R - 1)}{(\tau - 2)^2 (\tau + 1)(3\tau^2 + \tau - 6)^2} \leq 0 \); and \( \frac{\partial \Pi_{S_1}}{\partial C_R} - \frac{\partial \Pi_{S_2}}{\partial C_R} = \frac{2(\tau - 1)(\tau^2 - 2)(C_R - 1)}{(\tau - 2)^2 (\tau + 1)(3\tau^2 + \tau - 6)^2} \leq 0 \). Combining these conditions enables us to conclude that if \( \tau \leq \bar{\tau} = 0.54 \), we
have \( |\frac{\partial \Pi_{S_1}}{\partial C_R}| \geq |\frac{\partial \Pi_{S_2}}{\partial C_R}| \geq |\frac{\partial \Pi_R}{\partial C_R}| \geq |\frac{\partial \Pi_{A_1}}{\partial C_R}| = |\frac{\partial \Pi_{A_2}}{\partial C_R}| \); otherwise, we have \( |\frac{\partial \Pi_{S_1}}{\partial C_R}| \geq |\frac{\partial \Pi_R}{\partial C_R}| \geq |\frac{\partial \Pi_{A_1}}{\partial C_R}| = |\frac{\partial \Pi_{A_2}}{\partial C_R}| \). □

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