Risk2: Measuring the Risk in Value at Risk

Philippe Jorion

The recent derivative disasters have focused the attention of the finance industry on the need to control financial risks better. This search has led to a uniform measure of risk called value at risk (VAR), which is the expected worst loss over a given horizon at a given confidence level. VAR numbers, however, are themselves affected by sampling variation, or "estimation risk"—thus, the risk in value at risk itself. Nevertheless, given these limitations, VAR is an indispensable tool to control financial risks. This article lays out the statistical methodology for analyzing estimation error in VAR and shows how to improve the accuracy of VAR estimates.

The need to improve control of financial risks has led to a uniform measure of risk called value at risk (VAR), which the private sector is increasingly adopting as a first line of defense against financial risks. Regulators and central banks also provided the impetus behind VAR. The Basle Committee on Banking Supervision announced in April 1995 that capital adequacy requirements for commercial banks are to be based on VAR.1 In December 1995, the Securities and Exchange Commission issued a proposal that requires publicly traded U.S. corporations to disclose information about derivatives activity, with a VAR measure as one of three possible methods for making such disclosures. Thus, the unmistakable trend is toward more-transparent financial risk reporting based on VAR measures.

VAR summarizes the worst expected loss over a target horizon within a given confidence interval. VAR summarizes in a single number the global exposure to market risks and the probability of adverse moves in financial variables. It measures risk using the same units as the bottom line—dollars. Bankers Trust, for example, revealed in its 1994 annual report that its VAR was an average of $35 million at the 99 percent confidence level over one day; this number can be readily compared with its annual profit of $615 million or total equity of $4.7 billion. On the basis of such data, shareholders and managers can decide whether they feel comfortable with a level of risk. If the answer is no, the process that led to the computation of VAR can be used to decide where to trim risk.

In addition to financial reporting, VAR can be used for a variety of other purposes, such as setting position limits for traders, measuring returns on a risk-adjusted basis, and model evaluation. Institutional investors are also embracing VAR as a dynamic method for controlling their exposure to risk factors, especially when many outside fund managers are involved. Nonfinancial corporations, especially those involved with derivatives, are also considering risk-management systems centered around VAR. VAR provides a consistent measure of the effect of hedging on total risk, which is a significant improvement upon traditional hedging programs that typically focus only on individual transactions. No doubt these desirable features explain the wholesale trend toward VAR.

Current implementations of VAR, however, have not recognized the fact that VAR measures are only estimates of risk. VAR should be considered a first-order approximation to possible losses from adverse financial risk. Although VAR is a vast improvement over no measure at all, VAR numbers cannot be taken at face value. A VAR figure combines existing positions with estimates of risk (including correlations) over the target horizon. If these estimates are based on historical data, they inevitably will be affected by "estimation risk"; thus, value at risk also entails risk.2

Recognizing the existence of estimation risk has several important consequences. For instance, users might want to set the confidence level, usually set arbitrarily, to a value that will minimize the error in VAR. Or, the statistical methodology might be guided by the need to minimize estimation error.

In addition, VAR should be reported with confidence intervals. For instance, a bank might announce that its VAR over the next day is $35 million with a 95 percent confidence interval of $32 million to $38 million. A tight interval indicates relative con-

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fidence in the $35 million estimate, particularly compared with a hypothetical interval of $5 million to $65 million. The latter would say that the VAR number is quite inaccurate—although not in the range of billions. The purpose of this article is to provide a formal framework for analyzing estimation error in VAR and, more importantly, to discuss methods for improving the accuracy of VAR measures.

**MEASURING VAR**

To formally define a portfolio's VAR, one first must choose two quantitative factors: the length of the holding horizon and the confidence level. Both are arbitrary. As an example, the latest proposal of the Basle Committee defines a VAR measure using a 99 percent confidence interval over 10 trading days. The resulting VAR is then multiplied by a safety factor of 3 to arrive at the minimum capital requirement for regulatory purposes.

Presumably, the 10-day period corresponds to the time needed for regulators to detect problems and take corrective action. Presumably also, the choice of a 99 percent confidence level reflects the trade-off between the desire of regulators to ensure a safe and sound financial system and the adverse effect of capital requirements on bank profits. Different choices of horizon and confidence level will result in trivially different VAR numbers.

The significance of the quantitative factors depends on how they are to be used. If the resulting VARs are directly used for the choice of a capital cushion, then the choice of the confidence level is crucial. This choice should reflect the company's degree of risk aversion and the cost of a loss exceeding the VAR. Higher risk aversion, or greater costs, implies that a larger amount of capital should be available to cover possible losses, thus leading to a higher confidence level.

In contrast, if VAR numbers are used only to provide a companywide yardstick to compare risks among different markets, then the choice of confidence level is not very important. Assuming a normal distribution, disparate VAR measures are easy to convert into a common number.

To compute the VAR of a portfolio, define $W_0$ as the initial investment and $R$ as its rate of return. The portfolio value at the end of the target horizon is $W = W_0(1 + R)$. Define $\mu$ and $\sigma$ as the annual mean and standard deviation of $R$, respectively, and $\Delta t$ as the time interval considered. If successive returns are uncorrelated, the expected return and risk are then $\mu \Delta t$ and $\sigma \sqrt{\Delta t}$ over the holding horizon.

VAR is defined as the dollar loss relative to what was expected; that is,

$$VAR = E(W) - W^* = W_0(\mu - R^*),$$

where $W^*$ is the lowest portfolio value at given confidence level $c$. Finding VAR is equivalent to identifying the minimum value, $W^*$, or the cutoff return, $R^*$.

**VAR for General Distributions**

In its most general form, VAR can be derived from the probability distribution for the future portfolio value, $f(w)$. At a given confidence level, $c$, we wish to find the worst possible realization, $W^*$, such that the probability of exceeding this value is $c$, where

$$c = \int_{W^*}^{\infty} f(w) dw,$$

or such that the probability of a value lower than $W^*$ is $1 - c$, where

$$1 - c = \int_{W^*}^{\infty} f(w) dw.$$

In other words, the area from $\infty$ to $W^*$ must sum to $1 - c$, which might be, say, 5 percent. This specification is valid for any distribution, discrete or continuous, fat- or thin-tailed. As an example, in its 1994 annual report, J.P. Morgan revealed that its daily trading VAR averaged $15 million at the 95 percent level over one day. This number can be derived from Figure 1, which reports the distribution of J.P. Morgan's daily revenues in 1994.

From Figure 1, we find the average revenue is about $5 million. Next, we have to find the observation (also called a quantile) such that 5 percent of the distribution is on its left side. There are 254 observations, so we need to find $W^*$ such that the number of observations to its left is $254 \times 0.05 = 13$. This exercise yields $W^*$ equal to $-\$10 million and a daily VAR of $\$15 million.

**VAR for Normal Distributions**

If the distribution can be assumed to be normal, the computation can be simplified considerably. By using a multiplicative factor that is a function of the confidence level, VAR can be derived directly from the portfolio standard deviation.

First, map the general distribution $f(w)$ into a standard normal distribution $\Phi(\varepsilon)$, in which the random variable $\varepsilon$ has a mean of zero and a standard deviation of 1. The cutoff return, $R^*$, can be associated with a standard normal deviate $\alpha$ such that

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\[ -\alpha = \frac{\mu \Delta t - R^*}{\sigma \sqrt{\Delta t}}. \] (4)

Then, VAR may be found in terms of portfolio value \( W^* \), cutoff return \( R^* \), or normal deviate \( \alpha \); that is,

\[
1 - e = \int_{-\infty}^{\alpha} f(w) dw
\]

\[
R^* = \int_{-\infty}^{\alpha} f(r) dr
\]

\[
-\alpha = \int_{-\infty}^{\alpha} \Phi(\epsilon) d\epsilon.
\] (5)

To report VAR at the 95 percent confidence level, for example, the 5 percent left-tailed deviate from a standard normal distribution can be found from standard normal tables as 1.645. Once \( \alpha \) is identified, VAR can be recovered as

\[
\text{VAR} = W_0 \times \alpha \sigma \sqrt{\Delta t}.
\] (6)

The key result is that VAR is associated with the standard deviation only.

For instance, for the J.P. Morgan example from Figure 1, the standard deviation of the distribution is $9.2 million. Therefore, the normal-distribution VAR is

\[
\alpha(\sigma W_0) = 1.65 \times $9.2 \text{ million } = $15.2 \text{ million}.
\]

This number is very close to the VAR obtained from the general distribution, showing that the normal approximation provides a good estimate of VAR.

**Sigma-Based VAR**

Generally, this method applies to any probability function besides the normal, a convenient attribute because many financial variables have fatter tails (i.e., more extreme observations) than the normal distribution. Most notably, the stock market crash of October 1987 was a 20 standard deviation event—evident that under a normal distribution should never have happened. This behavior is particularly worrisome because VAR attempts to describe tail behavior precisely.

One possible explanation is that volatility changes through time, increasing in times of greater than normal turbulence. A stationary model might then erroneously view large observations as outliers when they are really drawn from a distribution with temporarily greater dispersion. Indeed, the recent literature on time variation in second moments provides overwhelming evidence that variances on a variety of financial assets do change over time.\(^3\)

Even controlling for time variation, however,
residual returns still appear to be fat tailed. One simple method to account for these tails is to model a Student t distribution, which is characterized by an additional parameter called “degrees of freedom” (v) that controls the size of tails. As v grows large, the distribution converges to a normal distribution.

Table 1 provides estimates of the Student parameter for a number of daily price returns over the 1990–94 period. Typically, the parameter v is in the range of 4 to 8, which confirms the existence of fat tails.

Table 1. Estimates of the Student Parameter

<table>
<thead>
<tr>
<th>Asset</th>
<th>Estimated Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. stocks</td>
<td>6.8</td>
</tr>
<tr>
<td>DM/US$ exchange rate</td>
<td>8.0</td>
</tr>
<tr>
<td>DM/E exchange rate</td>
<td>4.6</td>
</tr>
<tr>
<td>U.S. long bond</td>
<td>4.4</td>
</tr>
<tr>
<td>U.S. three-month T-bill</td>
<td>4.5</td>
</tr>
</tbody>
</table>

To find the VAR under a Student distribution, Equation 5 still applies, but \( \Phi \) is replaced by the standard Student distribution and \( \alpha \) by the appropriate \( 1 - c \) deviate. For example, for a Student with \( v \) equal to 6 at the 95 percent confidence level, \( \alpha \) equals 1.943.

Thus, for many distributions, the dispersion can be summarized by one parameter, the standard deviation. This approach applies to most financial prices, stock prices, bond prices, exchange rates, and commodities. Of course, it is inappropriate for strongly asymmetric distributions, such as positions in options. With large portfolios, such as trading portfolios of commercial banks, however, the issue is one of fatness in tails, not asymmetry.

**EVALUATING VAR**

So far, much of the analysis has been standard fare. What is less recognized, however, is the effect of estimation error. Indeed, all VAR measures are merely estimates. VAR measures are exact only when the underlying distribution is measured with an infinite number of observations. In practice, data are available for only a limited time period.

Various methods can be used to create VAR measures. “Historical-simulation” methods replicate the behavior of the current portfolio over a sample of previous days. “Delta-normal” methods summarize risk factors by a variance–covariance matrix estimated from historical data. J.P. Morgan’s RiskMetrics system, for instance, provides an application of the delta-normal method in which risk measures are time varying. In each case, using different time periods will invariably lead to different measures of VAR. The issue is whether sampling variation leads to material changes in VAR.

This possibility is why sensitivity analysis of VAR is a useful exercise. Beder (1995), for instance, compared VAR results from different models. Risk was measured at a two-week (10-day) horizon at the 5 percent level for a $1 million bond portfolio. Historical-simulation methods based on the previous 100 and 250 days yielded VARs of $2,000 and $17,000, respectively. The RiskMetrics method yielded a VAR of $18,200. These discrepancies appear to be wide and unsettling.

Upon further inspection, the historical-simulation method using 100 days appears to have been inadequate because of the small number of effective observations: only 10 (100 historical observations divided by the horizon of 10 business days). Such a small sample size makes the 5 percent left tail of a distribution difficult to measure. Otherwise, the remaining measures of VARs are in line with each other.

This experiment demonstrates the need for a good understanding of the methodology behind VAR. The question is whether discrepancies arise because of fundamental differences in methodologies or simply because of sampling variation.

**Estimation Error in Quantile-Based VAR**

For arbitrary distributions, the cth quantile can be empirically determined from the historical distribution as \( \tilde{q}(c) \). Of course, some sampling error is associated with the statistic. Kendall (1994), for instance, showed that the asymptotic standard error of the sample quantile, \( \tilde{q} \), is derived as

\[
se(\tilde{q}) = \frac{c(1-c)}{\sqrt{Tf(\tilde{q})}},
\]

where \( T \) is the sample size and \( f(\cdot) \) is the probability distribution function evaluated at the quantile \( \tilde{q} \). Kupiec (1995) pointed out that this standard error can be quite large and argued that this method does not provide “suitable benchmarks” for measuring VAR. In particular, the standard error increases markedly as the confidence level increases. In other words, the estimate grows increasingly unreliable farther into the left tail; that is, the 1 percent left tail is less reliable than the 10 percent quantile.

This phenomenon is illustrated in Figures 2 and 3, where the expected quantile and two standard error intervals are plotted for the normal and Student distributions, respectively.

For the normal distribution, the 5 percent left-tailed interval is centered around 1.645. With \( T \) equal to 100, a two standard error confidence interval is 1.24 to 2.04, which is quite large. With 250
observations, which correspond to one year of trading days, the interval is still 1.38 to 1.91. With $T$ equal to 1,000, the interval shrinks to 1.51 to 1.78. The interval widens substantially as one moves to more extreme quantiles. For instance, the same interval for the 1 percent quantile is 2.09 to 2.56 with $T$ equal to 1,000. As expected, more imprecision is found in the extreme tails, which have fewer data points.

Contrast these results with Figure 3, which displays confidence bands for a Student $t$ distribution.

**Figure 3. Confidence Bands for Sample Quantile: Student $t$ Distribution**

observations, which correspond to one year of trading days, the interval is still 1.38 to 1.91. With $T$ equal to 1,000, the interval shrinks to 1.51 to 1.78. The interval widens substantially as one moves to more extreme quantiles. For instance, the same interval for the 1 percent quantile is 2.09 to 2.56 with $T$ equal to 1,000. As expected, more imprecision is found in the extreme tails, which have fewer data points.

Contrast these results with Figure 3, which displays confidence bands for a Student $t$ distribution.

**Figure 3. Confidence Bands for Sample Quantile: Student $t$ Distribution**

Estimation Error in Sigma-Based VAR

Additional precision might be gained by directly measuring the standard deviation, which can be multiplied by an appropriate scaling factor to obtain the desired quantile.

For the normal distribution, for instance, the VAR can be computed in two steps: First, compute the sample standard deviation, $s$; then, multiply the number by a scaling factor, $\alpha(c)$, to obtain the desired confidence level—say, 1.645 for a 95 percent confidence level for a normal distribution.

Using this method, the standard error of the estimated quantile is

$$se(Q) = \alpha \times se(s).$$

(8)

This method leads to substantial efficiency gains relative to using the estimated quantile. In the case of the normal distribution, for instance, we know that the sample standard deviation is a sufficient statistic for the dispersion and also is the most efficient; that is, it has the lowest standard error. Intuitively, this efficiency is explained by the fact that $s$ uses information about the whole distribution (in terms of all squared deviations around the mean) but a quantile uses only the ranking of observations and the two observations around the estimated value.

For the normal distribution, we have an analytical formula for the standard error of $s$, which is

$$se(s|\Phi) = \sigma \sqrt{\frac{1}{2T}}.$$  

(9)

Figure 4 displays the standard error of the estimated quantile using the two methods applied to a normal distribution with $T$ equal to 250. As theory would suggest, the sample standard deviation method has uniformly lower standard errors and
is, therefore, uniformly superior to the sample quantile method.

A similar adjustment can be made for the Student \( t \) distribution, for which the quantile can be estimated from the sample standard deviation. Without analytical results for \( se(s) \), however, simulations must be used to determine the advantage of the \( s \)-based estimator over the usual quantile. Simulations are also useful for assessing the small-sample properties of these estimators.

**Comparisons of Methods**

Tables 2 and 3 describe VAR statistics from drawings from normal and Student \( t \) distributions, respectively, using simulations based on 10,000 replications. Two methods are compared: Method 1 is based on the sample quantile, and Method 2 is based on the sample standard deviation.

For Table 2 with one year of data (\( T = 250 \)), for example, the standard errors of VAR estimated using the two methods are generally in close accordance with the asymptotic numbers but the standard deviation method is about twice as efficient as the quantile method. In fact, the standard error of Method 1 is 0.133, as compared with 0.074 for Method 2, an improvement of about 45 percent.

The advantage is also substantial for the Student \( t \) distribution. For instance, using the same parameters as before, the standard error of Method 1 is 0.200 as opposed to 0.132 for Method 2, an improvement of about 35 percent. The advantage is even greater for quantiles farther in the tail, where the improvement is on the order of 60 percent.

**CONCLUSIONS**

The rapidly spreading use of VAR must be seen as a vast improvement over antiquated or nonexistent risk-management practices, some of which have caused financial disasters. One of the lessons of the Orange County, California, bankruptcy, for example, is that municipalities investing in the pool would have been more careful had the value at risk of their investment been clearly explained to them. In addition, investors would not have had the excuse that they did not know what they were getting into, which would have limited the rash of lawsuits against third parties. This experience demonstrates why regulators now embrace VAR as a means of improving transparency and stability in financial markets.

The benefits of VAR should not, however, mask its shortcomings. Any VAR number is itself measured with some error, or estimation risk. Thus, understanding the statistical methodology is important in order to interpret VAR estimates. This interpretation would be made easier not only by...
Table 2. VAR Statistics: Normal Distribution

<table>
<thead>
<tr>
<th>Statistic</th>
<th>$T = 100$</th>
<th>$T = 250$</th>
<th>$T = 1,000$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Simulation</td>
<td>Asymptotic</td>
<td>Simulation</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.998</td>
<td>0.071</td>
<td>0.071</td>
</tr>
<tr>
<td>VAR-Method 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quantile (10%)</td>
<td>-1.278</td>
<td>0.166</td>
<td>0.171</td>
</tr>
<tr>
<td>Quantile (5%)</td>
<td>-1.640</td>
<td>0.204</td>
<td>0.211</td>
</tr>
<tr>
<td>Quantile (1%)</td>
<td>-2.332</td>
<td>0.339</td>
<td>0.373</td>
</tr>
<tr>
<td>VAR-Method 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$ (10%)</td>
<td>-1.279</td>
<td>0.091</td>
<td>0.091</td>
</tr>
<tr>
<td>$\alpha$ (5%)</td>
<td>-1.641</td>
<td>0.117</td>
<td>0.116</td>
</tr>
<tr>
<td>$\alpha$ (1%)</td>
<td>-2.321</td>
<td>0.165</td>
<td>0.164</td>
</tr>
</tbody>
</table>

Notes: Averages and standard errors are based on 10,000 replications. Underlying distribution is the standard normal distribution $N(0,1)$. Asymptotic standard errors are analytically derived from exact parameters.
Table 3. VAR Statistics: Student t Distribution

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Simulation</th>
<th>Asymptotic</th>
<th>Simulation</th>
<th>Asymptotic</th>
<th>Simulation</th>
<th>Asymptotic</th>
<th>Exact</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T = 100</td>
<td></td>
<td>T = 250</td>
<td></td>
<td>T = 1,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>Standard Error</td>
<td>Average</td>
<td>Standard Error</td>
<td>Average</td>
<td>Standard Error</td>
<td></td>
</tr>
<tr>
<td>Standard deviation</td>
<td>1.216</td>
<td>0.128</td>
<td>NA</td>
<td></td>
<td>1.221</td>
<td>0.084</td>
<td>NA</td>
</tr>
<tr>
<td>VAR-Method 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quantile (10%)</td>
<td>-1.439</td>
<td>0.217</td>
<td>0.222</td>
<td>-1.439</td>
<td>0.139</td>
<td>0.140</td>
<td>-1.440</td>
</tr>
<tr>
<td>Quantile (5%)</td>
<td>-1.949</td>
<td>0.309</td>
<td>0.313</td>
<td>-1.977</td>
<td>0.200</td>
<td>0.198</td>
<td>-1.944</td>
</tr>
<tr>
<td>Quantile (1%)</td>
<td>3.267</td>
<td>0.848</td>
<td>0.781</td>
<td>-3.362</td>
<td>0.567</td>
<td>0.494</td>
<td>-3.148</td>
</tr>
<tr>
<td>VAR-Method 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a(10%)s</td>
<td>-1.430</td>
<td>0.151</td>
<td>NA</td>
<td>-1.436</td>
<td>0.098</td>
<td>NA</td>
<td>-1.439</td>
</tr>
<tr>
<td>a(5%)s</td>
<td>-1.926</td>
<td>0.205</td>
<td>NA</td>
<td>-1.934</td>
<td>0.132</td>
<td>NA</td>
<td>-1.939</td>
</tr>
<tr>
<td>a(1%)s</td>
<td>-3.118</td>
<td>0.329</td>
<td>NA</td>
<td>-3.131</td>
<td>0.214</td>
<td>NA</td>
<td>-3.138</td>
</tr>
</tbody>
</table>

NA = not available.

Notes: Averages and standard errors are based on 10,000 replications. Underlying distribution is a Student t distribution with 5 degrees of freedom. Asymptotic standard errors are analytically derived from exact parameters.
### Table 4. VAR Statistics: Empirical Distribution, Daily Return on 30-Year Bond, 1990–94

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Simulation $T = 100$</th>
<th></th>
<th>Simulation $T = 250$</th>
<th></th>
<th>Simulation $T = 1,000$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average</td>
<td>Standard Error</td>
<td>Average</td>
<td>Standard Error</td>
<td>Average</td>
<td>Standard Error</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.610</td>
<td>0.061</td>
<td>0.612</td>
<td>0.039</td>
<td>0.612</td>
<td>0.020</td>
</tr>
<tr>
<td>VAR–Method 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Quantile (10%)</td>
<td>-0.754</td>
<td>0.121</td>
<td>-0.757</td>
<td>0.078</td>
<td>-0.758</td>
<td>0.038</td>
</tr>
<tr>
<td>Quantile (5%)</td>
<td>-0.996</td>
<td>0.132</td>
<td>-1.010</td>
<td>0.078</td>
<td>-1.003</td>
<td>0.041</td>
</tr>
<tr>
<td>Quantile (1%)</td>
<td>-1.631</td>
<td>0.463</td>
<td>-1.643</td>
<td>0.313</td>
<td>-1.524</td>
<td>0.086</td>
</tr>
<tr>
<td>VAR–Method 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha(10%)$</td>
<td>-0.757</td>
<td>0.076</td>
<td>-0.759</td>
<td>0.049</td>
<td>-0.760</td>
<td>0.024</td>
</tr>
<tr>
<td>$\alpha(5%)$</td>
<td>-1.002</td>
<td>0.101</td>
<td>-1.005</td>
<td>0.065</td>
<td>-1.006</td>
<td>0.032</td>
</tr>
<tr>
<td>$\alpha(1%)$</td>
<td>-1.526</td>
<td>0.153</td>
<td>-1.530</td>
<td>0.098</td>
<td>-1.532</td>
<td>0.049</td>
</tr>
</tbody>
</table>

**Notes:** Averages and standard errors are based on 10,000 replications. Empirical distribution is bootstrapped from actual distribution of daily returns on the current 30-year bond from 1990 to 1994 (1,259 days).
reporting a single VAR number but also by reporting a confidence band around it.

The main purpose of this article was to show the extent to which VAR is affected by estimation risk and also to make recommendations on how to measure quantities. For the distributions considered here, estimating quantiles from a multiple of the sample standard deviation is far superior to estimating them directly from sample quantiles. Therefore, recognizing estimation error can lead to better measurement methods.

In the end, the greatest benefit of VAR may lie in the imposition of a structured methodology for thinking critically about risk. Financial institutions that go through the process of computing their VARs are forced to confront their exposure to financial risks and to set up a risk-management function to supervise the front and back offices. Thus, the process of getting to VAR may be as important as the number itself. Nevertheless, VAR is undoubtedly here to stay.

NOTES

1. The Basle Committee consists of central bankers from a group of 10 countries. This committee sets minimum standards for capital requirements in member countries.
2. In addition, for complex portfolios involving positions in options that are difficult to price, VAR is also affected by "model risk," which results from differences in valuations that can be traced to different option valuation models. Another conceptual problem is that, especially over long horizons, VAR does not account for changing positions. Sound risk-management practices typically reduce the size of positions in response to losses or increasing volatility, which decreases the worst loss relative to a static VAR measure. For further analysis of limitations of VAR methods, see Jorion (1996).
3. For a review, see, for instance, Bollerslev, Chou, and Kroner (1992). Time-series models that allow for time variation in risk can also capture structural changes as long as the changes do not occur too abruptly.
4. For instance, Bankers Trust uses a 99 percent level of confidence; Chemical Bank and Chase Manhattan, a 97.5 percent level; Citibank, a 95.4 percent level; and BankAmerica and J.P. Morgan, a 95 percent level.
5. These simulations assumed that the underlying distribution was normal. If the distribution is truly irregular because of, for example, heavy optionality in the portfolio, nonparametric methods such as kernel estimation can be used to provide estimates of the quantile and associated standard errors. These methods lead to improved precision by smoothing the distribution. See, for example, Sheather and Marron (1990).
6. For a description of the Orange County disaster, see Jorion (1995).

REFERENCES


