Enhanced Index Funds and Tracking Error Optimization

Philippe Jorion

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Correspondence should be addressed to:
Philippe Jorion,
University of California at Irvine
Graduate School of Management
Irvine, CA 92697-3125
E-mail: pjorion@uci.edu
(949) 824-5245

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Abstract

This paper tests the practical significance of the "tracking error optimization" theory. This predicts that active managers optimizing a portfolio subject to a constraint on tracking error will end up with higher total risk than the index. If so, measuring performance using the information ratio will give misleading results.

This conjecture is borne out by a sample of enhanced index funds in the U.S. tax-exempt market. We find that stock-based funds have systematically higher risk than the benchmark, as predicted by the theory. This, however, could also be achieved by leveraging up the benchmark. We show that, once this leverage effect is accounted for, this group of funds provides no better performance than the benchmark itself.

In contrast, derivatives-based enhanced index funds, which create value in another market and transport it to equities with derivatives, do not display this pattern of increased risk. In our sample, such funds add value after controlling for risk.

These results point to the dangers of judging performance solely by the information ratio. In addition to setting constraints on tracking error, investors should require managers to keep control of their total risk.

This research provides the first empirical assessment of the effect of optimization in excess returns on the risk profile of actively managed funds. It has become common practice to measure performance relative to the assigned benchmark, using tools such as "tracking risk" and the "information ratio." Institutional investors also routinely impose a limit on the volatility of the deviation of the active portfolio from the benchmark, also known as Tracking Error Volatility (TEV).

The problem with optimization in excess returns is that the manager totally ignores the investor's overall portfolio risk. In an insightful paper, Roll (1992) noted that portfolios optimized in this manner must systematically have higher risk than the benchmark and cannot be optimal in the absolute risk-return space.

To test the practical significance of this conjecture, we focus on "enhanced index funds", an industry that now exceeds $350 billion in assets under management. Such funds have become increasingly popular in recent years and are managed relative to a benchmark, attempting to maintain low TEV, typically below 4% per annum.

The growth of this industry, which has only blossomed in the last ten years, can be attributed to the success of index funds, which have proved hard to beat.[1] Presumably, enhanced index funds offer the best of both worlds: the risk profile of a highly-diversified portfolio and some value added due to active management. Relative to the usual active management style, enhanced index funds closely track the benchmark. As a result, enhanced index funds are now increasingly used as core equity portfolios that previously consisted of index funds. Stock-based enhanced funds are typically constructed with the tools of portfolio optimization, choosing stock positions so as to maximize expected returns subject to a TEV constraint. Thus, such funds provide a perfect experiment for investigating the effect of tracking error optimization.

Additionally, we investigate funds that use "alpha transportation" strategies, which involve adding value in another asset class and using derivatives to achieve equitization. Because such derivatives-based funds do not include the index assets

[1] There is ample literature that indicates that mutual funds as a whole do not outperform market indices. See Jensen (1968) and, more recently, Malkiel (1995) and Carhart (1997).

as part of the optimization, they should not necessarily have greater risk than the benchmark, in contrast to stock-based enhanced index funds.

This paper uses a database of U.S. tax-exempt funds classified as enhanced index funds, with the S&P 500 index as the benchmark. We provide evidence on the risk-return characteristics of such portfolios. In particular, we verify whether these active stock portfolios indeed have systematically higher volatility than the benchmark. If so, the crucial issue is whether the value added of such funds, if any, still appears on a risk-adjusted basis.

Focusing on enhanced index funds for performance measurement purposes has another important benefit. Compared to other active investment vehicles, it should also lead to more powerful statistical results due to the low tracking error. As a result, a shorter sample period should be required to establish statistical significance.

This line of research is increasingly important in view of the recent emphasis on risk control methods such as Value at Risk (VAR). VAR is a forward-looking measure of downside risk, based on current positions, and is becoming routinely used to measure and control risk.

The widespread implementation of VAR tools has led to the application of a "risk budgeting" concept to asset allocation.[2] Risk budgeting represents a process for converting efficient portfolio allocations into VAR assignments. Once the optimal portfolio is selected, the total fund risk is parceled out to managers in terms of VAR budgets, which account for diversification effects. Thus, active managers are increasingly given explicit VAR constraints, expressed in terms of deviations from the benchmark. Given some distributional assumption, these are equivalent to forward-looking TEV constraints.

This paper is structured as follows. Section 1 provides an introduction to indexing and the fundamental problem of performance measurement for active managers. Section 2 describes the growth of enhanced index funds and their different types. Section 3 then reviews the implications of the "tracking error optimization" theory

[2]See Chow and Kritzman (2001).

advanced by Roll. The data are described in Section 4 and analyzed in Section 5. Finally, Section 6 provides some concluding comments.

1 Indexing and Performance Measurement

The year 2000 marked the 30th anniversary of the first index fund, launched by Wells Fargo Bank in July 1971. It was a simple S&P 500 index fund, started with $6 million from the Samsonite Corporation. By now, 35 percent of assets in the U.S. pension fund industry are indexed, which amounts to approximately $1.5 trillion. The share of indexed pension fund assets in Europe is also increasing, at 25 percent in the U.K. and 13 percent in continental Europe.[3]

Similarly, in the mutual fund industry, it was only twenty-five years ago that the Vanguard 500 Index Fund was launched. Since then, this fund has become the world's largest mutual fund, with $96 billion in assets as of the end of 2000. Indexed funds now account for some $350 billion in mutual funds, which represents 8% of total assets.

The rising popularity of index funds can be squarely attributed to their excellent performance. Over the long run, index funds have outperformed their actively managed competitors, taken as a whole. In fact, this must be a mathematical truth. On average, active investors (defined as any other than passive investors) must obtain the market return, before management fees and trading costs. Once these costs are taken into account, actively managed funds must underperform indexed funds. Indeed, a number of studies show that mutual funds fail to provide aggregate returns that beat the market.

It is entirely possible, however, that some active managers, or class of managers, can consistently add value.[4] Outperformance in one way or another, unfortunately, is not easy to establish, due to the fact that statistical tests of performance have low

[3]See the *Financial Times*, May 17, 2001.

[4]If so, there should be some persistence in outperformance (at least before fees). There is some evidence of such repeat-winner phenomenon. See for instance Goetzmann and Ibbotson (1994) and Elton et al (1996).

power. In other words, it takes many years before one can be convinced, at usual statistical significance levels, that the manager indeed adds value.

Consider for instance a fund that provides a 4% expected return in excess of the benchmark. Suppose both the benchmark and active return have a volatility of 20% and a correlation of 0.8. This leads to a volatility of tracking error of 12.6%. To test whether the value added is statistically significant, we can use a t-statistic

$$t = \mu_\epsilon / (\sigma_\epsilon / \sqrt{T}) \tag{1}$$

where T is number of years, μ_ϵ is the average of excess return (added value), and σ_ϵ the standard deviation of tracking error, both expressed in annual terms.

Solving for T so that t is greater than the usual cutoff point of 2, we find that T must be greater than forty years. Unfortunately, this is longer than the average tenure of any portfolio manager. Waiting this long would surely tax the patience of any investor. If outperformance drops to 2% only, which is still a respectable track record, the period required is then 160 years!

Table 1 presents the required number of years for various combinations of value added and TEV. Decreasing the TEV drastically reduces the number of years required to wait for significance. With 4% value added and TEV=4%, we only need to wait 4 years to establish significance. Since enhanced index funds have tracking errors below 4%, it should be easier to establish skill for enhanced index fund managers.

TABLE 1 Number of Years Required for Significance

TEV	Value Added	Number of Years Required for Significance	Information Ratio
20%	4%	100	0.2
10%	4%	25	0.4
4%	4%	4	1.0
2%	4%	1	2.0
20%	2%	400	0.1
10%	2%	100	0.2
4%	2%	16	0.5
2%	2%	4	1.0

The previous reasoning was based on the tracking error, $\epsilon_j = R_j - R_b$, where R_j is the total return for portfolio j and b is the benchmark. This also applies to more general multifactor models such as

$$R_j = \alpha_j + \sum_k \beta_{jk} R_{bk} + \epsilon_j \quad (2)$$

where the statistical test applies to "alpha". With the market as the sole factor, and returns taken in excess of the risk-free rate, this specializes to Jensen's (1968) alpha.[5]

The table also reports the "information ratio" for various combinations. This is an adaptation of Sharpe's (1966) ratio, defined in terms of tracking error:

$$\omega = \mu_\epsilon / \sigma_\epsilon \quad (3)$$

The IR is commonly used to compare investment managers. Grinold and Kahn (1995), for example, assert that an IR of 0.50 is "good." Goodwin (1998) shows that an IR of 0.40 corresponds to the upper quartile for large-cap portfolios. Using the IR as a performance measure reflects a narrow focus on excess returns, which comes at the expense of ignoring total portfolio risk, however.

2 Enhanced Indexing

2.1 The Rise of Enhanced Indexing

Enhanced indexing primarily developed in response to the difficulty of beating the benchmark, in particular the S&P 500 index. Indeed, Lipper reports that, over the last 25 years, the average mutual fund has trailed the S&P 500 by 70bp annually. While this may not seem much, this difference compounds to large amounts over time.

In response to this poor performance, many firms have started to offer "enhanced" index funds. The purpose of such funds is to track the index closely, yet providing some value added. These funds have grown quickly in recent years.

[5] Performance evaluation tests, however, are subject to "model risk," due to uncertainty in the choice of the benchmark or in the number of factors in multiple-factor models. This is less of a problem with enhanced index funds, for which the benchmark is clearly identified and style attribution not necessary.

Indeed, surveys of fund managers of U.S. institutional tax-exempt assets indicate that, over the period 1994 to 2000, enhanced funds have grown from $33 to $365 billion, a ten-fold factor.[6] This is in contrast with passive indexed assets, which have grown more slowly. Enhanced assets currently account for 24% of total index assets for this group, which is quite substantial.

Another factor in favor of enhanced index funds is the greater control of risk they afford. Pension funds typically go through a strategic asset allocation process which carefully balances expected returns against risk, perhaps taking liabilities into account. The problem is that, once assets are assigned to active managers, large deviations from the benchmark may defeat this process. In contrast, enhanced index funds allow investors to maintain their desired risk profile.

2.2 Types of Enhanced Indexing

Enhanced indexing usually takes place in two forms. The first is based on cash markets, such as equities. Such funds invest solely in stocks, with careful under- or over-allocations to selected stocks. Managers maintain tight tracking error with quantitative risk control tools, such as quadratic optimization or control of exposure to multiple factors.[7]

More generally, these enhanced index funds can be viewed as a long position in the index plus a "hedge fund" with zero net investment, plus long and short positions in various securities, stocks or otherwise. With conventional active management, the assets are restricted to those in the universe, or the benchmark. Further, short positions in the hedge fund are only allowed up to the asset weight in the benchmark.

A second type of enhanced indexing is derivatives-based, or "synthetic". For instance, a exposure equivalent to the S&P 500 can be achieved by taking a long position in S&P stock index futures plus investing in cash. Alternatively, exposure can be achieved through positions in options or equity swaps. The manager then

[6] Williams, Fred, "Indexing Growth Slows in Last Half of 2000," *Pensions & Investments* (March 5, 2001).

[7] Chan et al. (1999), for instance, show that multi-factor models provide good control of tracking error risk.

adds value by increasing the return on cash, relative to the short-term risk-free rate. This can be achieved, for instance, by taking on some term structure risk, liquidity risk, credit risk, or mortgage-backed securities risk. This is sometimes called "alpha transportation strategies" since value is added in the bond market and then transported back to the stock market. One interesting issue is whether such funds have fundamentally different risk-return characteristics from cash-based enhanced index funds.

But this is not all. Many active funds are de facto becoming "closet" indexers. For instance, Morningstar reports that the average R-square of large cap active U.S. mutual funds with the S&P has increased from 0.71 to 0.86 from 1994 to 1999. Active managers must be increasingly tilting their portfolios toward the index. Thus enhanced index funds are only the most visible segment of a fundamental shift in asset management toward lower tracking error. This has profound implications for the asset management industry and performance measurement.

In practice, enhanced index managers use quantitative techniques to construct their portfolios. Typically, they use a Mean-Variance optimizer that yields the best risk-return tradeoff in an excess return space. This practice, however, runs into Roll's (1992) observation that such portfolios are not likely to be Mean-Variance efficient. So far, the Roll theory has not been tested, due to the fact that most active managers have high tracking error and that few perform explicit excess return optimization. With enhanced index funds, we now have a unique experiment to test whether this conjecture has practical implications.

3 Tracking Error Optimization

In a typical portfolio delegation problem, the investor assigns the management of assets to a portfolio manager who is given the task of beating a benchmark. When outperformance is observed for the active portfolio, the issue is whether the added value is in line with the risks undertaken. This is particularly important with performance fees, which induce an option-like pattern in the compensation of the agent, who has an incentive to take on more risk to increase the value of the option.

To control for this, institutional investors commonly impose a limit on the volatility of the deviation of the active portfolio from the benchmark, also known as Tracking Error Volatility (TEV), which can also be expressed in terms of Value-at-Risk. The TEV constraint is typically very tight for enhanced index funds, around 2% annually.

The problem with focusing on excess returns with a TEV constraint is that it induces the manager to optimize in an excess return space only, totally ignoring the investor's overall portfolio risk. Consider a universe of N assets, which include the benchmark. For instance, the manager may choose among 5,000 stocks, which include the 500 in the index. The active portfolio consists of index weights q plus active deviations x.

Let us define E as the vector of expected returns, and V as the covariance matrix for asset returns. The efficient set constants are $a = E'V^{-1}E, b = E'V^{-1}1, c = 1'V^{-1}1$, where 1 represents a vector of ones. The constant $\sqrt{d}$, where $d = a - b^2/c$, is also the slope of the hyperbola's asymptotes, or maximal information ratio achievable by the active manager with this set of assets. We define the variance of tracking error as σ_x^2.

The portfolio is optimized with two constraints, one on TEV and the other on total deviations adding up to zero. Roll (1992) shows that the vector x of deviations from the benchmark can be written as

$$x = \frac{\sigma_x}{\sqrt{d}}\; V^{-1}[E - \frac{b}{c}] \tag{4}$$

which, surprisingly, does not depend on the benchmark.[8]

Further, in this setup, the total portfolio must have higher risk than the benchmark. Generally, we have

$$\sigma_P^2 = \sigma_B^2 + \sigma_x^2 + 2\sigma_B\sigma_x\rho_{x,B} \tag{5}$$

where σ_B is the benchmark's volatility and $\rho_{x,B}$ the correlation.

[8] See also Jorion (2002) for detailed derivations.

Using the deviations from (4) and the fact that the covariance matrix includes the benchmark assets, we can decompose the total portfolio risk as

$$\sigma_P^2 = \sigma_B^2 + \sigma_x^2 + 2\sigma_x \frac{1}{\sqrt{d}}(\mu_B - \frac{b}{c}) \tag{6}$$

where μ_B is the expected return on the benchmark. We suppose that $\mu_B > b/c$, otherwise the benchmark would be totally dominated by the global minimum-variance portfolio. As a result, $\rho_{x,B}$ must be positive. Thus the total portfolio risk must increase due to the addition of the second term σ_x^2, which is always positive, and to the third term, which is also positive when resulting from an excess return optimization.

This is not necessarily the case, however, when the optimization is performed over a universe of assets that does not include the index assets. The total risk is then

$$\sigma_P^2 = \sigma_B^2 + \sigma_x^2 + 2q'V_{q,x}x \tag{7}$$

where $V_{q,x}$ is the covariance matrix between the index assets and the optimized ones. In a derivatives-based strategy, the value added comes from another set of assets, such as the fixed-income market, which typically has low correlation with equities. In such a case, the third term should be close to zero. Thus the risk profile of stock-based funds, which involve an explicit optimization of the type described by Roll, should be different from that of derivatives-based funds.

We can develop further insights by writing (6) as

$$\sigma_P^2 = \sigma_B^2 + \sigma_x^2 + 2\sigma_x\sigma_B[\frac{(\mu_B - b/c)/\sigma_B}{\sqrt{d}}] \tag{8}$$

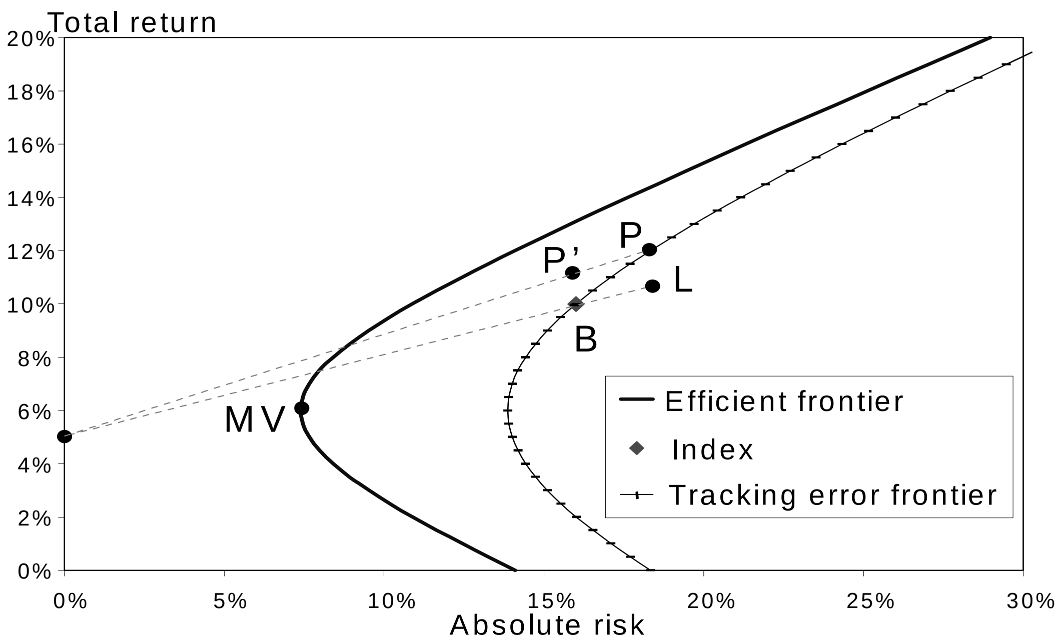
where the last term between brackets, which is also the correlation coefficient, represents the efficiency of the benchmark relative to that of the assets.[9] As an illustration, assume this ratio is 0.5, and take $\sigma_B = 16\%$ and $\sigma_x = 2\%$. The portfolio risk is then 17.1%; with no correlation, it would be only 16.1%. With a tracking error of 4%, the total risk is 18.3% and 16.5% with and without correlation. This correlation is therefore the main driver of the increase in risk.

[9] This is similar to the measure developed by Kandel and Stambaugh (1995) and represents the ratio of the slope of the line from the global minimum-variance portfolio to the benchmark to the slope of the asymptote.

The increase in risk is illustrated in Figure 1, which displays the TEV frontier in the original mean-standard deviation plane. The TEV frontier must go through the benchmark B, since the manager has the option to have zero TEV, and thus zero value added. Point P represents the active portfolio.

The graph shows an unintended effect of tracking error optimization: Instead of moving toward the true efficient frontier, i.e. up and to the left of the index, the tracking error frontier moves up and to the right. The reason why this result is significant is that the increased volatility could also be achieved by leveraging up the benchmark, which should also increase the return.

FIGURE 1 Effect of TEV Optimization



In this example, P outperforms the benchmark by 2%. Part of this higher performance, unfortunately, can be attained by increasing the risk of the benchmark up to the level of the active portfolio. This is described by portfolio L, which has the same total risk as the active portfolio. Here, portfolio L is 73 basis points above the benchmark due to the leverage effect. This is a non-negligible fraction of the 200 basis points in excess performance of the active portfolio.

This leverage effect can be corrected by a Sharpe ratio measure. Equivalently, a risk-adjusted performance measure (RAP_P) can be defined as

$$RAP_P = R_f + (\bar{R}_P - R_f)(\sigma_P/\sigma_B)^{-1} \tag{9}$$

where R_f is the risk-free rate, $\bar{R}_P$ is the average portfolio return, and σ_P/σ_B is the ratio of the volatility of the portfolio to that of the benchmark. Here, this is 18.3%/16%=1.1455. The performance for portfolio P' represents RAP_P in Figure 1. The risk-adjusted excess return is

$$RAER_P = RAP_P - \bar{R}_B \tag{10}$$

Here, the adjustment gives $RAP_P = 5\% + (10\% - 5\%)/1.1455 = 11.11\%$, for a risk-adjusted excess return of $RAER_P = 11.11\% - 10.00\% = 1.11\%$ instead of the original 2.00%. In this case, 45% of the value added for the active portfolio P is fallacious.

Thus, the Roll conjecture is a puzzling result, especially in view of the widespread practice of imposing limits on tracking error volatility. Are investors unaware of this problem, or is it not a problem? If it is observed in practice, how much of the outperformance could be explained by this leverage effect?

4 Data

The database for this study consists of performance data provided by Pensions & Investments Performance Evaluation Reports (PIPER), published by Pensions & Investments (P&I). This source reports quarterly total rates of return, before fees, for a universe of pools managed for U.S. tax exempt funds going back to 1990. The advantage of this data source is that enhanced indexation is much more widespread and explicit among institutional managers than mutual funds. Asset managers in this industry routinely use quantitative techniques such as portfolio optimization and multi-factor risk systems.

Enhanced equity funds are identified through a two-pass process. We start from the list of top managers of enhanced index assets provided by P&I every year from 1995 to 2000. For all the managers on this list, we then search through funds whose names contain the key words "enhanced", "structured", "alpha tilt", "optimized", "plus", or "core equity". As a double-check, we only keep funds that closely track the S&P 500 index, with less than 4% annual tracking error volatility over the period 1996-2000.

Table 2 displays the list of enhanced index fund managers for which we have return data. The second column reports total tax-exempt U.S. assets classified as enhanced assets, which total $363 billion as of December 2000. This amount includes bonds as well. The third column reports total enhanced funds in U.S. equities, which total $277 billion. When one firm offers various funds, the active fund with lowest tracking error was selected. Finally, the last column gives the size of the enhanced stock index funds in our sample.[10]

Table 3 describes the 45 funds in the analysis. Whenever possible, we report the strategy followed by the fund. Out of the 45 funds in our sample, 29 are cash-based, investing in stocks directly. Of the remainder, 14 funds use derivatives-based strategies involving futures, swaps, options, or index-based arbitrage.

[10]Note that there may be some inconsistencies in the data for the same firm, due to erratic reporting.

TABLE 2 Assets of Enhanced Index Funds Managers (Dec-2000)

	U.S. Tax-exempt assets ($m)		
Firm	Enhanced Assets	Of which, US Equities	Fund used
Advanced Investment Mgt	$2,664	$2,505	$1,873
Aeltus Investment Mgt	$9,246		$6,481
Alliance Capital Mgt	$4,417		$741
Amalgamated Bank	$127		$127
American Express			$45
Barclays Global Investors	$60,600	$37,600	$13,439
Brinson Partners			$686
Caterpillar Invst Mgt			$65
Chicago Equity Partners			$270
Clifton Group			$151
Common Fund			$175
Credit Suisse Invest Mgt	$448		$364
Deutsche Asset Mgt	$1,891	$1,555	$1,400
DSI International Mgt	$5,100	$5,138	$4,100
Fidelity Investments	$14,551	$17,904	$7,350
Fiduciary Asset Mgt		$1,378	$1,016
First Capital Group	$1,133		$1,133
First Quadrant Corp		$1,692	$650
Franklin			$1,054
Goldman Sachs Asset Mgt		$2,274	$9,980
Independence			$6,864
INTECH	$5,364		$3,279
INVESCO	$1,231	$2,355	$1,631
Jacobs Levy Equity Mgt			$1,823
JP Morgan Investment Mgt	$53,349	$45,684	$26,350
Lotsoff Capital Mgt	$240	$240	$189
Mellon Capital Mgt		$2,767	$573
Mellon Equity Associates			$35
Metropolitan West Asset	$680		$606
McGahan GreeneMcHugh Cap		$1,281	$1,281
Monitor Capital Advisors	$13		$41
Noddings Investmt Group			$150
Northern Trust Glob Inv	$217		$64
Oakbrook Investments	$52		$41
Pacific Investment Mgt	$18,106	$18,069	$16,342
Pangora Asset Mgt			$113
Parametric Portfolio Ass		$1,167	$1,723
Payden & Rygel		$1,297	$800
Prudential Investment	$4,594	$3,782	$1,840
Rampart Investment Mgt			$51
Smith Breeden Associates		$1,638	$1,829
State Street Global Advisors	$16,390	$3,012	$1,215
Weiss Peck			$556
Westpeak Investment Advisors		$2,236	$1,573
Westridge Capital Mgt	$1,100		$1,300
Subtotal	$201,513	$153,574	$121,369
Total, with others	$363,367	$276,615	

Source: Pensions & Investments March 19, 2001 for total assets, May 14, 2001 for U.S. equities, and PIPER database for individual funds.

TABLE 3 Enhanced Index Funds

Firm	Fund Name	Period: First Quarter	Strategy Assets	Strategy Process	Mgt. Fee (bp)
Advanced	Enhanced S&P500	Mar-91	Bonds	Deriv	
Aeltus	Enhanced Core Equity	Dec-91	Stocks	Risk control	35
Alliance Capital	Enhanced Passive S&P500	Sep-92	Bonds	Futures	
Amalgamated Bank	Long View Quant Large Cap	Mar-97	Stocks	Risk control	51
American Express	Research Core	Mar-90	Stocks	Top-down	20
Barclays GI	Alpha Tilts	Mar-90	Stocks	Risk control	
Brinson	Structured S&P500 Portf	Mar-90	Stocks	Risk control	20
Caterpillar	Enhanced S&P500	Dec-95	Bonds	Deriv	20
Chicago Equity	Large Cap Enhanced Equity	Mar-96	Stocks	Multifactor	
Clifton Group	Enhanced S&P Composite	Mar-90	Bonds	Futures	30
Common Fund	Index Plus Fund	Mar-92	Various	Swap	
Credit Suisse	Structured Core Equity	Jun-90	Stocks	Risk control	
Deutsche Asset	Pyramid Enhanced S&P500	Sep-96	Stocks	Multifactor	8
DSI	US Controlled Residuals	Mar-96	Stocks	Quantitative	25
Fidelity	Select Large Cap	Dec-90	Stocks	Risk control	35
Fiduciary Asset	Enhanced Index/Core	Dec-95	Stocks	Optimizer	40
First Capital	Enhanced Stock Fund	Mar-90	NA		
First Quadrant	Enhanced Index	Mar-93	Stocks	Multifactor	43
Franklin	Large Cap Active	Mar-90	Stocks	Multifactor	19
Goldman Sachs	Core S&P500 STE	Mar-90	Stocks	Multifactor	40
Independence	Diversified Core	Mar-90	Stocks	Multifactor	45
INTECH	Large Cap Core	Mar-90	Stocks	Risk control	45
INVESCO	Risk Controlled Alpha	Mar-90	Stocks	Multifactor	44
Jacobs Levy	Core Equity S&P500	Sep-91	Stocks	Multifactor	69
JP Morgan	Research Enhanced Index	Mar-90	Stocks	Risk control	
Lotsoff Capital	COREX-Enhanced Index	Sep-94	Bonds	Futures	30
Mellon Capital	Stock Performance Fund	Sep-91	Bonds	Futures	8
Mellon Equity	Lg Cap S&P500 Enhanced	Mar-96	Stocks	Multifactor	25
Metropol.West	Alpha Trak	Dec-97	Bonds	Futures	
McGahan Greene	Large Cap Diversified Core	Sep-91	Stocks	Risk control	25
Monitor Capital	Enhanced Index-Large Cap	Mar-98	Stocks	Risk control	
Noddings	S&P500 Enhanced	Sep-96	Bonds	Futures	
Northern Trust	NTQA Enhanced S&P500	Mar-95	Stocks	Multifactor	40
Oakbrook	Enhanced Index Strategy	Mar-95	Stocks	Risk control	24
Pacific	StocksPLUS	Mar-90	Bonds	Futures	35
Pangora	Large Cap US Equity	Mar-90	Stocks	Optimization	45
Parametric	Active Core	Mar-90	NA		
Payden & Rygel	Enhanced Equity	Jun-94	Bonds	Futures	25
Prudential	Quantitative Core Equity	Sep-95	Stocks	Multifactor	25
Rampart Investment	Enhanced Index	Jun-99	Bonds	Options	
Smith Breeden	Equity Market Plus	Sep-92	Bonds	Futures	35
State Street	Large Cap Index Plus	Jun-93	Stocks	Multifactor	35
Weiss Peck	Large Cap Disciplined	Mar-90	Stocks	Multifactor	
Westpeak	Enhanced S&P500 Strategy	Mar-93	Stocks	Multifactor	15
Westridge	Enhanced Index Equity	Mar-90	Bonds	Index arb/fut	
				Average:	32

Six funds only cover less than five years; nineteen funds have at least ten year of data. The average period does not differ much across strategies, with 8 years of data for stock-base funds, and 7.5 years for derivatives-based funds.

The table also records, whenever available, annual management fees for each fund. The reported fees are the lowest available, for large accounts. For the whole sample, fees average out to 32bp; fees are slightly higher, at 34bp for the cash-based group, versus 26bp for derivatives-based funds. These fees are much lower than those for mutual funds, which average more than 100bp. On the other hand, totally passive index funds have rock-bottom fees, about 6bp.

Because the database only contains survivors, in existence at the end of the sample, the aggregate performance of the industry will be overstated. This has to be kept in mind for the interpretation of aggregate performance measures for this group of funds. Malkiel, for example, reports a 10-year mortality rate of 20% among mutual funds, which causes a bias of 140bp in average returns in his sample. Carhart et al. (2001) report an annual attrition rate of 3.6% per year. Like Goetzmann and Jorion (1999), they show that the bias is increasing with the sample period length. Over a ten-year period, their survival bias is approximately 0.70%. Brown, Goetzmann and Ross (1995), however, show that survivorship does create any particular bias in volatility measures.

5 Empirical Analysis

Table 4 describes the cross-section of performance numbers for excess returns on the sample of enhanced index funds. The top half describes 1996-2000 data; the bottom half covers 1991-2000. The 5-year panel uses 39 firms with exactly five years of data and six with slightly shorter periods. The 10-year panel has 19 firms with exactly ten years of data and others with less.

The sample is also separated into 29 stock-based funds and 14 derivatives-based funds. For the former, we assume a common management fee of 34bp, versus 26bp for the latter.

TABLE 4 Cross-Section of Excess Return Performance

	Mean	Min	25th	Median	75th	Max	Percent	P-val
Period: 1996-2000, 45 funds								
Average Return	0.54	-2.40	0.02	0.70	1.05	2.76	75.6%> 0	0.0001
Std.Dev. (TEV)	2.05	0.50	1.09	2.07	2.90	3.81		
Information Ratio	0.48	-0.66	0.01	0.36	0.72	4.79	75.6%> 0	0.0001
I.R., after fee	0.27	-0.76	-0.18	0.19	0.54	4.34	64.4%> 0	0.0178
T-statistic	1.07	-1.49	0.01	0.73	1.54	10.70	15.6%> 2	0.0016
T-stat, after fee	0.61	-1.70	-0.41	0.43	1.12	9.70	11.1%> 2	0.0239
Period: 1996-2000, 29 stock-based funds								
Average Return	0.59	-2.40	0.02	0.80	1.14	2.62	75.9%> 0	0.0012
Std.Dev. (TEV)	2.33	0.79	1.61	2.28	3.10	3.81		
Information Ratio	0.38	-0.66	0.01	0.36	0.63	1.50	75.9%> 0	0.0012
I.R., after fee	0.20	-0.76	-0.08	0.22	0.50	1.16	69.0%> 0	0.0121
T-statistic	0.83	-1.49	0.01	0.73	1.42	3.36	13.8%> 2	0.0136
T-stat, after fee	0.44	-1.70	-0.19	0.49	1.11	2.58	10.3%> 2	0.0548
Period: 1996-2000, 14 derivatives-based funds								
Average Return	0.60	-1.79	0.12	0.53	0.90	2.76	85.7%> 0	0.0009
Std.Dev. (TEV)	1.53	0.50	0.65	1.09	2.25	3.80		
Information Ratio	0.81	-0.52	0.08	0.67	0.88	4.79	85.7%> 0	0.0009
I.R., after fee	0.55	-0.60	-0.08	0.29	0.63	4.34	64.3%> 0	0.0898
T-statistic	1.82	-0.69	0.19	1.41	1.98	10.70	21.4%> 2	0.0042
T-stat, after fee	1.24	-0.80	-0.17	0.65	1.41	9.70	14.3%> 2	0.0301
Period: 1991-2000, 45 funds								
Average Return	0.74	-1.79	0.32	0.82	1.15	3.81	82.2%> 0	0.0000
Std.Dev. (TEV)	2.05	0.42	1.18	2.07	2.81	4.41		
Information Ratio	0.49	-0.52	0.23	0.48	0.68	1.78	82.2%> 0	0.0000
I.R., after fee	0.29	-0.60	-0.01	0.30	0.49	1.46	73.3%> 0	0.0004
T-statistic	1.36	-1.28	0.66	1.19	1.93	5.64	24.4%> 2	0.0000
T-stat, after fee	0.79	-1.65	-0.02	0.82	1.46	4.63	11.1%> 2	0.0239
Period: 1991-2000, 29 stock-based funds								
Average Return	0.79	-0.58	0.59	0.89	1.22	2.19	82.8%> 0	0.0001
Std.Dev. (TEV)	2.24	0.79	1.57	2.20	2.90	3.92		
Information Ratio	0.43	-0.19	0.23	0.47	0.58	1.44	82.8%> 0	0.0001
I.R., after fee	0.25	-0.35	0.09	0.28	0.48	1.01	79.3%> 0	0.0003
T-statistic	1.15	-0.60	0.66	1.09	1.85	3.22	17.2%> 2	0.0027
T-stat, after fee	0.66	-0.96	0.27	0.72	1.40	2.27	6.9%> 2	0.1751
Period: 1991-2000, 14 derivatives-based funds								
Average Return	0.78	-1.79	0.34	0.79	0.96	3.81	85.7%> 0	0.0009
Std.Dev. (TEV)	1.74	0.42	1.01	1.36	2.41	4.41		
Information Ratio	0.69	-0.52	0.30	0.70	1.09	1.78	85.7%> 0	0.0009
I.R., after fee	0.45	-0.60	0.08	0.50	0.82	1.46	71.5%> 0	0.0287
T-statistic	2.01	-0.69	0.86	1.83	2.83	5.64	42.9%> 2	0.0000
T-stat, after fee	1.34	-0.90	0.20	1.26	1.83	4.63	21.4%> 2	0.0042

Notes: The table presents statistics for the cross-section of performance numbers for enhanced index funds in excess of the S&P500 index. Data are in percent per annum. The effective sample period varies across funds, as reported in Table 3. The last column reports the percentage of average returns and information ratios above zero and of t-statistics above 2, as well as p-values that the fraction is significant.

The top panel shows that the tracking error volatility ranges from 0.5 to 3.8%, with a median of 2.0%, which is indeed very tight.[11] The information ratio ranges from -0.66 to 4.79, with a median of 0.36. After fees, the median information ratio falls to 0.19, which is still positive but not very impressive. The top quartile IR is 0.54 after fees, in line with the view that an IR above 0.5 is "good".

The t-statistic tests the hypothesis of zero outperformance. We find that 15.6%, or 7 of the funds have a t-statistic above 2. Of course, we could expect some significant numbers simply by chance. Assuming independent observations, we can describe the distribution of t-statistics under the null of 5% type I error with a binomial distribution. This distribution has expected value of 2.25; observing more than 4 exceedences has a p-value of 0.07. The p-value for 7 (15.6%) or more such funds is only 0.0016. After fees, we find 5 (11.1%) funds with a t-statistic above 2, which is significant. This suggests active management skills. We need to bear in mind, however, that this sample is subject to survivorship biases.

Similar results hold for the 10-year sample. Over a longer period, information ratios increase, to a median of 0.48 before fees and 0.30 after fees. Thus, there seems to be some value added.

The next panels break down the sample into stock-based and derivatives-based funds. Over the 1996-2000 period, the first group has a median TEV of 2.28%, with value added of 0.80%. The second group has lower median TEV of 1.09%, with slightly lower value added, 0.53%.

Adjusting for tracking risk, the median IR after fees of the first group is 0.22. That of the second group is slightly higher, at 0.29. The difference is greater over the 10-year sample, with median IR after fees of 0.28 an 0.50, respectively. On average, both groups seem to be adding value.

Next, Table 5 describes the cross-section of risk characteristics of enhanced index funds. As before, the top half describes 1996-2000 data, and the bottom half 1991-2000 data.

[11] As a reference, the Mellon EB Stock Index Fund, a passive fund with $33 billion in assets, had tracking error risk of 0.09% only over this period. Frino and Gallagher (2001) investigate sources of tracking error for S&P 500 mutual funds and report a range of 0.05 to 0.12%.

The last columns give the fraction of funds with betas greater than unity, as well as tests of significance. Over 1996-2000, about 69% of funds have a beta greater than unity, which is significant. Even a larger fraction has greater volatility than the benchmark. From the "volatility ratio" line, we see that this is the case for 73% of the funds. As explained before, this must be due to a positive correlation between benchmark deviations and index return. The next line indeed shows that the correlation $\rho_{x,B}$ is generally positive.

TABLE 5 Risk Characteristics of Enhanced Index Funds

	Mean	Min	25th	Median	75th	Max	Percent	Nb.	P-val
Period: 1996-2000, 45 funds									
Beta	1.010	0.859	0.989	1.013	1.037	1.117	68.9%> 1	31	0.0033
Volatility Ratio	1.019	0.879	0.997	1.015	1.050	1.132	73.3%> 1	33	0.0004
Corr-TE with S&P	0.090	-0.667	-0.114	0.197	0.281	0.580	68.9%> 0	31	0.0033
Period: 1996-2000, 29 stock-based funds									
Volatility Ratio	1.033	0.954	1.013	1.028	1.057	1.132	82.8%> 1	24	0.0001
Corr-TE with S&P	0.162	-0.560	0.087	0.224	0.288	0.580	75.9%> 0	22	0.0012
Period: 1996-2000, 14 derivatives-based funds									
Volatility Ratio	0.988	0.879	0.963	1.002	1.013	1.071	50.0%> 1	7	0.3953
Corr-TE with S&P	-0.084	-0.667	-0.491	-0.006	0.233	0.465	50.0%> 0	7	0.3953
Period: 1991-2000, 45 funds									
Beta	1.006	0.884	0.989	1.011	1.030	1.106	66.7%> 1	30	0.0080
Volatility Ratio	1.037	0.900	0.994	1.021	1.056	1.470	73.3%> 1	33	0.0004
Corr-TE with S&P	0.080	-0.556	-0.042	0.100	0.259	0.580	73.3%> 0	33	0.0004
Period: 1991-2000, 29 stock-based funds									
Volatility Ratio	1.036	0.900	1.011	1.021	1.050	1.284	79.3%> 1	23	0.0003
Corr-TE with S&P	0.127	-0.556	0.079	0.114	0.259	0.580	79.3%> 0	23	0.0003
Period: 1991-2000, 14 derivatives-based funds									
Volatility Ratio	1.041	0.929	0.972	1.007	1.054	1.470	57.1%> 1	8	0.2120
Corr-TE with S&P	-0.032	-0.538	-0.300	0.033	0.182	0.465	57.1%> 0	8	0.2120

Notes: The table describes the cross-section of risk characteristics of enhanced index funds. "Beta" is the systematic risk relative to the S&P 500. "Volatility ratio" is the ratio of total risk of the fund relative to that of the S&P, measured over the same period. "Corr-TE with S&P" describes the correlation of the tracking error with the return on the S&P. The last columns report the fraction of betas and volatility ratios above one, of correlation above zero, as well as p-values under the null.

Next, we investigate whether this pattern is similar across stock-based and derivatives-based funds. The table shows that stock-based enhanced index funds systematically display greater risk than the benchmark. The fraction is 83%, which is significant. On the other hand, for derivatives-based strategies, the fraction is exactly 50%. Assuming a baseline market volatility of 16%, the average volatility of stock-based funds is 16.5%, versus 15.8% for the other group.

The average correlation between the excess return and the benchmark is positive for stock-based funds, and negative for derivatives-based funds. As shown in Equation (5), this correlation is an important driver of total risk. Since the excess return in derivatives-based funds appears orthogonal, or even has a negative correlation with the benchmark, the total risk of these funds is, on average, lower than the benchmark. This supports the conjecture that the effect of tracking error optimization is most pronounced for stock-based enhanced index funds.

Table 6 presents risk-adjusted performance data. Across all funds over 1996-2000, the mean Risk-Adjusted Excess Return (RAER) is slightly positive, at 0.29%. Once fees are factored in, however, the average performance falls to zero. The median is still slightly positive, at 0.23%. Similar result are obtained with Jensen's alpha. We have 4 funds (8.9% of the sample) for which the t-statistic exceeds 2. With a p-value of 0.0729, however, this number is not significantly different from the 5% we would expect from a truly random sample.

Next, we separate out the different types of funds. For stock-based enhanced index funds, there is little evidence of outperformance. After fees, the RAER has a mean of -0.17% and median of 0.06%. Exactly half of the sample has positive performance. This is quite different from the results we obtained from the information ratio analysis, which suggested skill. Here, after adjusting for increased volatility, there remains no evidence of skill (even in a sample that may be affected by survivorship).

Derivatives-based funds, however, tell a different story. After fees, the mean RAER is 0.45%, and the median is 0.39%. Out of the 14 funds, 11 (78.6% of the sample) have positive performance, which is significant.

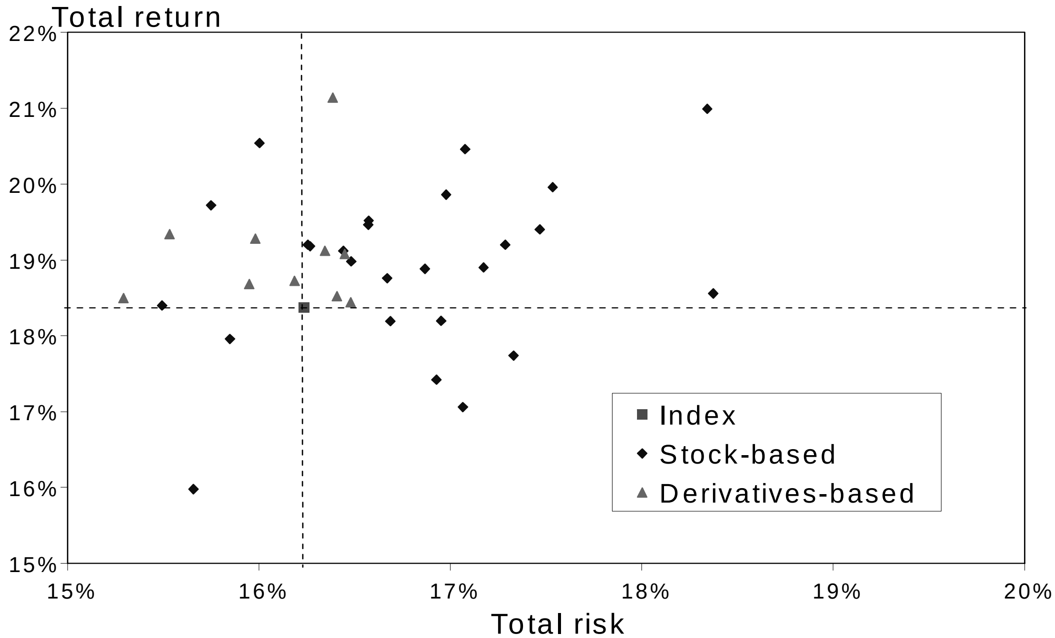
TABLE 6 Performance Characteristics of Enhanced Index Funds

	Mean	Min	25th	Median	75th	Max	Percent	Nb.	P-val
Period: 1996-2000, 45 funds									
Risk-Adj.Exc.Ret.	0.29	-2.04	-0.13	0.52	0.84	2.62	66.7%> 0	30	0.0080
RAER-after fee	-0.02	-2.34	-0.44	0.23	0.50	2.36	57.8%> 0	26	0.1856
Alpha	0.38	-1.93	-0.01	0.63	0.99	2.65	73.3%> 0	33	0.0004
Alpha-after fee	0.07	-2.19	-0.35	0.30	0.65	2.39	60.0%> 0	27	0.0676
T-stat, net alpha	0.46	-1.65	-0.70	0.39	0.77	8.57	8.9%> 2	4	0.0729
Period: 1996-2000, 29 stock-based funds									
RAER	0.17	-2.00	-0.23	0.40	0.80	2.39	62.1%> 0	18	0.0680
RAER-after fee	-0.17	-2.34	-0.57	0.06	0.46	2.05	51.7%> 0	15	0.3555
Period: 1996-2000, 14 derivatives-based funds									
RAER	0.71	-2.04	0.42	0.65	1.26	2.62	85.7%> 0	12	0.0009
RAER-after fee	0.45	-2.30	0.16	0.39	1.00	2.36	78.6%> 0	11	0.0065
Period: 1991-2000, 45 funds									
Risk-Adj.Exc.Ret.	0.32	-3.15	-0.03	0.58	0.87	2.78	73.3%> 0	33	0.0004
RAER-after fee	0.01	-3.49	-0.37	0.28	0.61	2.52	62.2%> 0	28	0.0362
Alpha	0.57	-1.93	0.34	0.77	0.92	3.16	82.2%> 0	37	0.0000
Alpha-after fee	0.35	-1.53	0.01	0.42	0.85	2.01	75.6%> 0	34	0.0001
T-stat, net alpha	0.47	-2.16	0.02	0.59	1.06	3.53	4.4%> 2	2	0.3923
Period: 1991-2000, 29 stock-based funds									
RAER	0.30	-3.15	0.17	0.58	0.75	1.60	75.9%> 0	22	0.0012
RAER-after fee	-0.04	-3.49	-0.17	0.24	0.41	1.26	62.1%> 0	18	0.0680
Period: 1991-2000, 14 derivatives-based funds									
RAER	0.52	-2.09	-0.02	0.85	1.25	2.78	71.4%> 0	10	0.0287
RAER-after fee	0.26	-2.35	-0.28	0.59	0.99	2.52	71.4%> 0	10	0.0287

Notes: The table describes the cross-section of performance data of enhanced index funds. Risk-Adjusted Excess Returns (RAER) are based on Equation (10), which adjusts the performance for the level of volatility vis-a-vis the benchmark. "Alpha" is the usual Jensen measure, which adjusts for beta risk. Both measures are presented before and after fees. "T-stat, net alpha" gives the t-test of significance for alpha, net of fees. The last columns give the percentage of numbers greater than zero and associated tests.

These results are summarized in Figure 2, which plots total return against total risk for the funds for which we have full data over 1996-2000. The graph clearly shows that stock-based funds have much greater volatility, on average, than derivatives-based funds. This confirms the "tracking error optimization" theory.

FIGURE 2 Total Return and Risk: 1996-2000



In view of these results, the fact that TEV constraints are widely used in the industry is puzzling. Roll (1982) conjectured that "a tracking error goal is appropriate for individual portfolio managers because fund sponsors usually employ an entire stable of managers."

The volatility characteristics of the portfolio are driven by two factors, the number of funds as well as the correlation of their excess returns. With sufficiently low correlations, ideally negative, we should observe a sharp fall in TEV as the number of funds increases. Ideally, the portfolio total risk should move closer to that of the index.

To see the effects of diversification on TEV, assume for simplicity that all active returns have the same TEV σ_x and correlation ρ. The portfolio TEV can be written as

$$\sigma[\frac{1}{N}\sum_{i=1}^{N} R_{x,i}] = \sigma_x\sqrt{\frac{1}{N} + (1 - \frac{1}{N})\rho}. \quad (11)$$

The top half of Table 7 summarizes these average correlation coefficients, measured over the period 1996-2000. For all 45 funds, the average correlation is 0.095, with 61 percent positive. Thus these correlations are not really low, considering their effect on the total TEV in (11). For instance, with 20 funds and zero correlation, the TEV decreases to 22% of the individual TEV; with a correlation of 0.095, this drops to only 37%.

The average correlation across cash-based funds is even higher, at 0.122, which indicates that many funds pursue similar strategies. Across derivatives-based funds, the average correlation is 0.100, which suggests again similar strategies. Average correlations across cash-based and derivatives-based funds, however, are lower, at 0.048.

The bottom half of the table explores implications for portfolios of funds. Recall that the average fund has volatility ratio of $\sigma_P/\sigma_B = 1.019$ and tracking error of 2.05%. We need 19 funds to halve the TEV to 0.90%, which is a large number of funds. In addition, the total volatility ratio is hardly changed, at 1.018.

With 39 funds, the TEV drops further to 0.70%. The volatility ratio drops to 1.010. Thus, the total volatility drops slightly, but is still above the index. This suggests that employing a stable of managers is not likely to correct the deficiencies of tracking error optimization, unless the number of managers is very high.

TABLE 7 Portfolios of Enhanced Index Funds

Period: 1996-2000

Funds	Average Correlation of Excess Return
All 45 funds	0.095
All 29 stock-based funds	0.122
All 14 deriv.-based funds	0.100
Between 29 and 14 funds	0.048

	Volatility	Volatility Ratio	TEV
S&P 500 Index	16.23%	1.000	0.00
Average Fund	16.54%	1.019	2.05
19-Fund Portfolio	16.53%	1.018	0.90
39-Fund Portfolio	16.41%	1.010	0.70

Notes: The table describes the performance of portfolio of funds over the period 1995-2000. The top half reports the average correlations among groups of funds. The bottom half compares the total volatility and tracking error volatility (TEV) for the index, for the average fund, for a 19-fund portfolio and 39-fund portfolio.

6 Conclusions

This paper has tested the practical significance of the "tracking error optimization" theory. This predicts that active managers optimizing a portfolio subject to a TEV constraint will structure a portfolio that has higher total risk than the index. This is not necessarily the case for derivatives-based strategies.

We showed that this conjecture is borne out by the data. The characteristics of our sample of stock-based enhanced index funds sharply differ from derivatives-based funds.

Stock-based funds implement small tilts away from the positions in the benchmark subject to constraints on the TEV. We find that 83% of the funds do have higher risk than the benchmark. As predicted by the theory, most portfolios have positive correlations between tracking error and the index. Their average information ratio is, after fees, about 0.20. Focusing on the IR only, and ignoring survivorship, this groups seems to add value.

This result, however, is reversed when adjusting for the higher total risk of these funds. They deliver on average a risk-adjusted excess return of 0.17% before fees, but -0.17% after fees. There is no evidence that this group of enhanced index funds beats the benchmark. This is in large part due to the increased risk of these active managers.

Our conclusions differ for derivatives-based funds. These funds generate value-added in some other market, for example fixed-income, then transport this alpha back to U.S. equities through positions in stock index derivatives. Such structures are less susceptible to problems inherent in tracking error optimization. Indeed we find that such funds have, as a whole, a risk profile similar to that of the benchmark. They deliver on average a risk-adjusted excess return of 0.71% before fees, and 0.45% after fees. On average, these funds add value.

The main message of this paper is that tracking error optimization has perverse effects, as predicted. It does seem to increase the risk profile of active stock managers. The historical evidence in this paper is that the value added is fallacious as it represents increased risk. If investors can accept this increased risk profile, they would be better served by leveraging up the benchmark. In the meantime, they should insist that performance be measured in terms of total risk. The widespread emphasis in the industry on information ratios is misleading.

On a more positive note, active managers could easily alter their risk profile since they already have an optimizer in place. Investors could require additional constraints beyond the usual TEV limit. Jorion (2002) derives analytical solutions for the risk-return relationship of portfolios subject to a TEV constraint. This relationship is described by an ellipse in the mean-variance plane. Due to the flat shape of this ellipse, adding a constraint on total portfolio volatility can substantially improve risk-adjusted performance. Given that current technology allows managers to measure their risk on an ex ante basis, it would be easy to incorporate such restrictions into the portfolio optimization process.

References

Brown, Stephen J., William Goetzmann, and Stephen Ross, 1995, "Survival," *Journal of Finance* 50, 853–873.

Carhart, Mark, 1997, "On Persistence in mutual fund performance," *Journal of Finance* 52 (March), 57–82.

Carhart, Mark, Jennifer Carpenter, Anthony Lynch, and David Musto, 2001, "Mutual fund survivorship," mimeo, *New York University*.

Chan, L.K., Karceski, J. and Josef Lakonishok, 1999 "On Portfolio Optimization: Forecasting Covariances and Choosing the Risk Model," *Review of Financial Studies* 12 (Winter): 937–974.

Chow, George and Mark Kritzman, 2001, "Risk Budgets," *Journal of Portfolio Management* 27 (Winter): 56–64.

Elton, Edwin J., Gruber, Martin J., and Blake, Christopher R., 1996, "The Persistence of Risk-Adjusted Mutual Fund Performance," *Journal of Business* 69 (April): 133–157.

Frino, Alex and David Gallagher, 2001, "Tracking S&P 500 Index Funds," *Journal of Portfolio Management* 28 (Fall): 44–55.

Goetzmann, William N., and Ibbotson, Roger G., 1994, "Do Winners Repeat? Patterns in Mutual Fund Return Behavior," *The Journal of Portfolio Management* 20 (Winter): 9–18.

Goetzmann, William and Philippe Jorion, 1999, "Re-emerging Markets," *Journal of Financial and Quantitative Analysis* 34 (March): 1–32.

Goodwin, Thomas, 1998, "The Information Ratio," *Financial Analysts Journal* 4 (July): 34–43.

Grinold, Richard and Ronald Kahn, 1995, *Active Portfolio Management*, Chicago: Irwin.

Jensen, Michael, 1968, "The Performance of Mutual Funds during the Period 1945-64," *Journal of Finance* 23 (May): 389–416.

Jorion, Philippe, 2002, *Portfolio Optimization with Constraints on Tracking Error,* mimeo.

Kandel, Shmuel, and Robert Stambaugh, 1995, "Portfolio Inefficiency and the Cross-Section of Expected Returns," *Journal of Finance* 50 (March): 157–184.

Malkiel, Burton G., 1995. "Returns from Investing in Equity Mutual Funds 1971 to 1991," *Journal of Finance* 50 (June): 549–572.

Roll, Richard, 1992, "A Mean-Variance Analysis of Tracking Error," *Journal of Portfolio Management* 18(4) (Summer): 13–22.

Sharpe, William, 1966, "Mutual Fund Performance," *Journal of Business* 39 (January): 119–38.