

## Explaining the Rate Spread on Corporate Bonds

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### ABSTRACT

The purpose of this article is to explain the spread between rates on corporate and government bonds. We show that expected default accounts for a surprisingly small fraction of the premium in corporate rates over treasuries. While state taxes explain a substantial portion of the difference, the remaining portion of the spread is closely related to the factors that we commonly accept as explaining risk premiums for common stocks. Both our time series and cross-sectional tests support the existence of a risk premium on corporate bonds.

THE PURPOSE OF THIS ARTICLE is to examine and explain the differences in the rates offered on corporate bonds and those offered on government bonds (spreads), and, in particular, to examine whether there is a risk premium in corporate bond spreads and, if so, why it exists.

Spreads in rates between corporate and government bonds differ across rating classes and should be positive for each rating class for the following reasons:

1. Expected default loss—some corporate bonds will default and investors require a higher promised payment to compensate for the expected loss from defaults.
2. Tax premium—interest payments on corporate bonds are taxed at the state level whereas interest payments on government bonds are not.
3. Risk premium—The return on corporate bonds is riskier than the return on government bonds, and investors should require a premium for the higher risk. As we will show, this occurs because a large part of the risk on corporate bonds is systematic rather than diversifiable.

The only controversial part of the above analyses is the third point. Some authors in their analyses assume that the risk premium is zero in the corporate bond market.<sup>1</sup>

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<sup>1</sup> Many authors assume a zero risk premium. Bodie, Kane, and Marcus (1993) assume the spread is all default premium. See also Fons (1994) and Cumby and Evans (1995). On the other hand, rating-based pricing models like Jarrow, Lando, and Turnbull (1997) and Das-Tufano (1996) assume that any risk premium impounded in corporate spreads is captured by adjusting transition probabilities.

This paper is important because it provides the reader with explicit estimates of the size of each of the components of the spread between corporate bond rates and government bond rates.<sup>2</sup> Although some studies have examined losses from default, to the best of our knowledge, none of these studies has examined tax effects or made the size of compensation for systematic risk explicit. Tax effects occur because the investor in corporate bonds is subject to state and local taxes on interest payments, whereas government bonds are not subject to these taxes. Thus, corporate bonds have to offer a higher pre-tax return to yield the same after-tax return. This tax effect has been ignored in the empirical literature on corporate bonds. In addition, past research has ignored or failed to measure whether corporate bond prices contain a risk premium above and beyond the expected loss from default (we find that the risk premium is a large part of the spread). We show that corporate bonds require a risk premium because spreads and returns vary systematically with the same factors that affect common stock returns. If investors in common stocks require compensation for this risk, so should investors in corporate bonds. The source of the risk premium in corporate bond prices has long been a puzzle to researchers and this study is the first to provide both an explanation of why it exists and an estimate of its importance.

Why do we care about estimating the spread components separately for various maturities and rating classes rather than simply pricing corporate bonds off a spot yield curve or a set of estimated risk neutral probabilities? First, we want to know the factors affecting the value of assets and not simply their value. Second, for an investor thinking about purchasing a corporate bond, the size of each component for each rating class will affect the decision of whether to purchase a particular class of bonds or whether to purchase corporate bonds at all.

To illustrate this last point, consider the literature that indicates that low-rated bonds produce higher average returns than bonds with higher ratings whereas the lower-rated bonds do not have a higher standard deviation of return.<sup>3</sup> What does this evidence indicate for investment? This evidence has been used to argue that low-rated bonds are attractive investments. However, we know that this is only true if required return is no higher for low-rated debt. Our decomposition of corporate spreads shows that the risk premium increases for lower-rated debt. In addition, because promised coupon is higher for lower-rated debt, the tax burden is greater. Thus, the fact that lower-rated bonds have higher realized returns does not imply they are better investments because the higher realized return might not be sufficient compensation for taxes and risk.

<sup>2</sup> Liquidity may play a role in the risk and pricing of corporate bonds. We, like other studies, abstract from this influence.

<sup>3</sup> See, for example, Altman (1989), Goodman (1989), Blume, Keim, and Patel (1991), and Cornell and Green (1991).

The paper proceed as follows: in the first section we start with a description of our sample. We next discuss both the need for using spot rates (the yield on zero-coupon bonds) to compute spreads and the methodology for estimating them. We examine the size and characteristics of the spreads. As a check on the reasonableness of the spot curves, we estimate, for government and corporate bonds, the ability of our estimated spot rates to price bonds. The next three sections (Sections II–IV) of the paper present the heart of our analysis: the decomposition of rate spreads into that part which is due to expected loss, that part which is due to taxes, and that part which is due to the presence of systematic risk.

In the first of these sections (Sec. II), we model and estimate that part of the corporate spread which is due to expected default loss. If we assume for the moment that there is no risk premium, then we can value corporate bonds under the assumption that investors are risk neutral using expected default losses.<sup>4</sup> This risk neutrality assumption allows us to construct a model and estimate what the corporate spot rate spread would be if it were solely due to expected default losses. We find that the spot rate spread curves estimated by incorporating only the expected default losses are well below the observed spot spread curve and that they do not increase as we move to lower ratings as fast as actual spot spread curves. In fact, expected loss can account for no more than 25 percent of the corporate spot spreads.

In Section III, we examine the impact of both the expected default loss and the tax premium on corporate spot spreads. In particular, we build both expected default loss and taxes into the risk neutral valuation model developed earlier and estimate the corporate spot rates that should be used to discount promised cash payments when both state and local taxes and expected default losses are taken into consideration. We then show that using the best estimate of tax rates, actual corporate spot spreads are still much higher than what taxes and default premiums can together account for.

Section IV presents direct evidence of the existence of a risk premium and demonstrates that this risk premium is compensation for the systematic nature of risk in bond returns. We first relate the time series of that part of the spreads that is not explained by expected loss or taxes to variables that are generally considered systematic priced factors in the literature of financial economics. Then we relate cross-sectional differences in spreads to sensitivities of each spread to these variables. We have already shown that the default premium and tax premium can only partially account for the difference in corporate spreads. In this section we present direct evidence that there is a premium for systematic risk by showing that the majority of the corporate spread, not explained by defaults or taxes, is explained by factor sensitivities and their prices. Further tests suggest that the factor sensitivities are not proxies for changes in expected default risk.

Conclusions are presented in Section V.

<sup>4</sup> We also temporarily ignore the tax disadvantage of corporate bonds relative to government bonds in this section.

## I. Corporate Yield Spreads

In this section, we examine corporate yield spreads. We initially discuss the data used. Then we discuss why yield spreads should be measured as the difference in yield to maturity on zero-coupon bonds (rather than coupon bonds) and how these rates can be estimated. Next, we examine and discuss the pattern of spreads. Finally, we compare the price of corporate bonds computed from our estimated spots with actual prices as a way of judging the reasonableness of our estimates.

### A. Data

Our bond data are extracted from the Lehman Brothers Fixed Income Database distributed by Warga (1998). This database contains monthly price, accrued interest, and return data on all investment-grade corporate and government bonds. In addition, the database contains descriptive data on bonds, including coupons, ratings, and callability.

A subset of the data in the Warga database is used in this study. First, all bonds that were matrix priced rather than trader priced are eliminated from the sample.<sup>5</sup> Employing matrix prices might mean that all our analysis uncovers is the rule used to matrix-price bonds rather than the economic influences at work in the market. Eliminating matrix-priced bonds leaves us with a set of prices based on dealer quotes. This is the same type of data as that contained in the standard academic source of government bond data: the CRSP government bond file.<sup>6</sup>

Next, we eliminate all bonds with special features that would result in their being priced differently. This means we eliminate all bonds with options (e.g., callable bonds or bonds with a sinking fund), all corporate floating rate debt, bonds with an odd frequency of coupon payments, government flower bonds, and inflation-indexed government bonds.

In addition, we eliminate all bonds not included in the Lehman Brothers bond indexes, because researchers in charge of the database at Lehman Brothers indicate that the care in preparing the data was much less for bonds not included in their indexes. This results in eliminating data for all bonds with a maturity of less than one year.

<sup>5</sup> For actively traded bonds, dealers quote a price based on recent trades of the bond. Bonds for which a dealer did not supply a price have prices determined by a rule of thumb relating the characteristics of the bond to dealer-priced bonds. These rules of thumb tend to change very slowly over time and to not respond to changes in market conditions.

<sup>6</sup> The only difference in the way CRSP data is constructed and our data is constructed is that over the period of our study, CRSP uses an average of bid/ask quotes from five primary dealers called randomly by the New York Federal Reserve Board rather than a single dealer. However, comparison of a period when CRSP data came from a single dealer and also from the five dealers surveyed by the Fed showed no difference in accuracy (Sarig and Warga (1989)). Also in Section II, we show that the errors in pricing government bonds when spots are extracted from the Warga data are comparable to the errors when spots are extracted from CRSP data. Thus our data should be comparable in accuracy to the CRSP data.

Finally, we eliminate bonds where the price data or return data was problematic. This involved examining the data on bonds that had unusually high pricing errors when priced using the spot curve. Bond pricing errors were examined by filtering on errors of different sizes and a final filter rule of \$5 was selected.<sup>7</sup> Errors of \$5 or larger are unusual, and this step resulted in eliminating 2,710 bond months out of our total sample of 95,278 bond months. Examination of the bonds that are eliminated because of large differences between model prices using estimated spots and recorded prices show that large differences were caused by the following:

1. The price was radically different from both the price immediately before the large error and the price after the large error. This probably indicates a mistake in recording the data.
2. The company issuing the bonds was going through a reorganization that changed the nature of the issue (such as interest rate or seniority of claims), and this was not immediately reflected in the data shown on the tape, and thus the trader was likely to have based the price on inaccurate information about the bond's characteristics.
3. A change was occurring in the company that resulted in the rating of the company to change so that the bond was being priced as if it were in a different rating class.

### *B. Measuring Spreads*

Most previous work on corporate spreads has defined corporate spread as the difference between the yield to maturity on a coupon-paying corporate bond (or an index of coupon-paying corporate bonds) and the yield to maturity on a coupon-paying government bond (or an index of government bonds) of the same maturity.<sup>8</sup> We define spread as the difference between yield to maturity on a zero-coupon corporate bond (corporate spot rate) and the yield to maturity on a zero-coupon government bond of the same maturity (government spot rate). In what follows we will use the name "spot rate" rather than the longer expression "yield to maturity on a zero-coupon bond" to refer to this rate.

The basic reason for using spots rather than yield to maturity on coupon debt is that arbitrage arguments hold with spot rates, not with yield to maturity. Because a riskless coupon-paying bond can always be expressed as

<sup>7</sup> The methodology used to do this is described later in this paper. We also examined \$3 and \$4 filters. Employing a \$3 or \$4 filter would have eliminated few other bonds, because there were few intermediate-size errors, and we could not find any reason for the error when we examined the few additional bonds that would be eliminated.

<sup>8</sup> The prices in the Warga Database are bid prices as are the bond price data reported in DRI or Bloomberg. Because the difference in the bid and ask price in the government market is less than this difference in the corporate market, using bid data would result in a spread between corporate and government bonds even if the price absent the bid/ask spread were the same. However, the difference in price is small and, when translated to spot yield differences, is negligible.

a portfolio of zeros, spot rates are the rates that must be used to discount cash flows on riskless coupon-paying debt to prevent arbitrage.<sup>9</sup> The same is not true for yield to maturity. In addition, the yield to maturity depends on coupon. Thus, if yield to maturity is used to define the spread, the spread will depend on the coupon of the bond that is picked. Finally, calculating spread as difference in yield to maturity on coupon-paying bonds with the same maturity means one is comparing bonds with different duration and convexity.

The disadvantage of using spots is that they need to be estimated.<sup>10</sup> In this paper, we use the Nelson–Siegel procedure (see Appendix A) for estimation of spots. This procedure was chosen because it performs well in comparison to other procedures.<sup>11</sup>

### *C. Empirical Spreads*

The corporate spread we examine is the difference between the spot rate on corporate bonds in a particular rating class and spot rates for Treasury bonds of the same maturity. Table I presents Treasury spot rates as well as corporate spreads for our sample for the three following rating classes: AA, A, and BBB for maturities from two to ten years. AAA bonds were excluded because for most of the 10-year period studied, the number of these bonds that existed and were dealer quoted was too small to allow for accurate estimation of a term structure of spots. Corporate bonds rated below BBB were excluded because data on these bonds was not available for most of the time period we studied.<sup>12</sup> Initial examination of the data showed that the term structure for financials was slightly different from the term structure for industrials, and so in this section, the results for each sector are reported separately.<sup>13</sup> In Panel A of Table I, we have presented the average difference over our 10-year sample period, 1987 to 1996. In Panels B and C we present similar results for the first and second half of our sample period. We expect these differences to vary over time.

<sup>9</sup> Spot rates on promised payments may not be a perfect mechanism for pricing risky bonds because the law of one price will hold as an approximation when applied to promised payments rather than risk-adjusted expected payments. See Duffie and Singleton (1999) for a description of the conditions under which using spots to discount cash flows is consistent with no arbitrage.

<sup>10</sup> The choice between defining spread in terms of yield to maturity on coupon-paying bonds and spot rates is independent of whether we include matrix-priced bonds in our estimation. For example, if we use matrix-priced bonds in estimating spots we will improve estimates only to the extent that the rules for matrix pricing accurately reflect market conditions.

<sup>11</sup> See Nelson and Siegel (1987). For comparisons with other procedures, see Green and Odegaard (1997) and Dahlquist and Svensson (1996). We also investigated the McCulloch cubic spline procedure and found substantially similar results throughout our analysis. The Nelson and Siegel model was fit using standard Gauss–Newton nonlinear least squares methods.

<sup>12</sup> We use both Moody's and S&P data. To avoid confusion we will always use S&P classifications, though we will identify the sources of data. When we refer to BBB bonds as rated by Moody's, we are referring to the equivalent Moody's class, named Baa.

<sup>13</sup> This difference is not surprising because industrial and financial bonds differ both in their sensitivity to systematic influences and to idiosyncratic shocks that occurred over the time period.

**Table I**  
**Measured Spread from Treasury**

This table reports the average spread from treasuries for AA, A, and BBB bonds in the financial and industrial sectors. For each column, spot rates were derived using standard Gauss-Newton nonlinear least square methods as described in the text. Treasuries are reported as annualized spot rates. Corporates are reported as the difference between the derived corporate spot rates and the derived treasury spot rates. The financial sector and the industrial sector are defined by the bonds contained in the Lehman Brothers' financial index and industrial index, respectively. Panel A contains the average spot rates and spreads over the entire 10-year period. Panel B contains the averages for the first five years and panel C contains the averages for the final five years.

Maturity	Treasuries	Financial Sector			Industrial Sector		
		AA	A	BBB	AA	A	BBB
Panel A: 1987–1996							
2	6.414	0.586	0.745	1.199	0.414	0.621	1.167
3	6.689	0.606	0.791	1.221	0.419	0.680	1.205
4	6.925	0.624	0.837	1.249	0.455	0.715	1.210
5	7.108	0.637	0.874	1.274	0.493	0.738	1.205
6	7.246	0.647	0.902	1.293	0.526	0.753	1.199
7	7.351	0.655	0.924	1.308	0.552	0.764	1.193
8	7.432	0.661	0.941	1.320	0.573	0.773	1.188
9	7.496	0.666	0.955	1.330	0.589	0.779	1.184
10	7.548	0.669	0.965	1.337	0.603	0.785	1.180
Panel B: 1987–1991							
2	7.562	0.705	0.907	1.541	0.436	0.707	1.312
3	7.763	0.711	0.943	1.543	0.441	0.780	1.339
4	7.934	0.736	0.997	1.570	0.504	0.824	1.347
5	8.066	0.762	1.047	1.599	0.572	0.853	1.349
6	8.165	0.783	1.086	1.624	0.629	0.872	1.348
7	8.241	0.800	1.118	1.644	0.675	0.886	1.347
8	8.299	0.813	1.142	1.659	0.711	0.897	1.346
9	8.345	0.824	1.161	1.672	0.740	0.905	1.345
10	8.382	0.833	1.177	1.682	0.764	0.912	1.344
Panel C: 1992–1996							
2	5.265	0.467	0.582	0.857	0.392	0.536	1.022
3	5.616	0.501	0.640	0.899	0.396	0.580	1.070
4	5.916	0.511	0.676	0.928	0.406	0.606	1.072
5	6.150	0.512	0.701	0.948	0.415	0.623	1.062
6	6.326	0.511	0.718	0.962	0.423	0.634	1.049
7	6.461	0.510	0.731	0.973	0.429	0.642	1.039
8	6.565	0.508	0.740	0.981	0.434	0.649	1.030
9	6.647	0.507	0.748	0.987	0.438	0.653	1.022
10	6.713	0.506	0.754	0.993	0.441	0.657	1.016

There are a number of interesting results reported in this table. Note that, in general, the corporate spread for a rating category is higher for financials than it is for industrials. For both financial and industrial bonds, the corporate

spread is higher for lower-rated bonds for all spots across all maturities in both the 10-year sample and the 5-year subsamples. Bonds are priced as if the ratings capture real information. To see the persistence of this influence, Figure 1 presents the time pattern of spreads on 6-year spot payments for AA, A, and BBB industrial bonds month by month over the 10 years of our sample. Note that the curves never cross. A second aspect of interest is the relationship of corporate spread to the maturity of the spot rates. An examination of Table I shows that there is a general tendency for the spreads to increase as the maturity of the spot lengthens. However, for the 10 years from 1987 to 1996, and each 5-year subperiod, the spread on BBB industrial bonds exhibits a humped shape.

The results we find can help differentiate among the corporate debt valuation models derived from option pricing theory. The upward sloping spread curve for high-rated debt is consistent with the models of Merton (1974), Jarrow, Lando, and Turnbull (1997), Longstaff and Schwartz (1995), and Pitts and Selby (1983). It is inconsistent with the humped shape derived by Kim, Ramaswamy and Sundaresan (1987). The humped shape for BBB industrial debt is predicted by Jarrow et al. (1997) and Kim et al. (1987), and is consistent with Longstaff and Schwartz (1995) and Merton (1974) if BBB is considered low-rated debt.<sup>14</sup> However, one should exercise care in interpreting these results, for, as noted by Helwege and Turner (1999), the tendency of less risky companies within a rating class to issue longer-maturity debt might tend to bias yield and to some extent spots on long maturity bonds in a downward direction.

We will now examine the results of employing spot rates to estimate bond prices.

#### *D. Fit Error*

One test of our data and procedures is to see how well the spot rates extracted from coupon bond prices explain those prices. We do this by directly comparing actual prices with the model prices derived by discounting coupon and principal payments at the estimated spot rates. Model price and actual price can differ because of errors in the actual price and because bonds within the same rating class, as defined by a rating agency, are not homogenous. We calculate model prices for each bond in each rating category every month using the spot yield curves estimated for that rating class in that month. For each month, average error (error is measured as actual minus model price) and the square root of the average squared error are calculated. These are then averaged over the full 10 years and separately for the first and last 5 years for each rating category. The average error for all

<sup>14</sup> While the BBB industrial curve is consistent with the models that are mentioned, estimated default rates shown in Table IV are inconsistent with the assumptions these models make. Thus, the humped BBB industrial curve is inconsistent with spread being driven only by defaults.

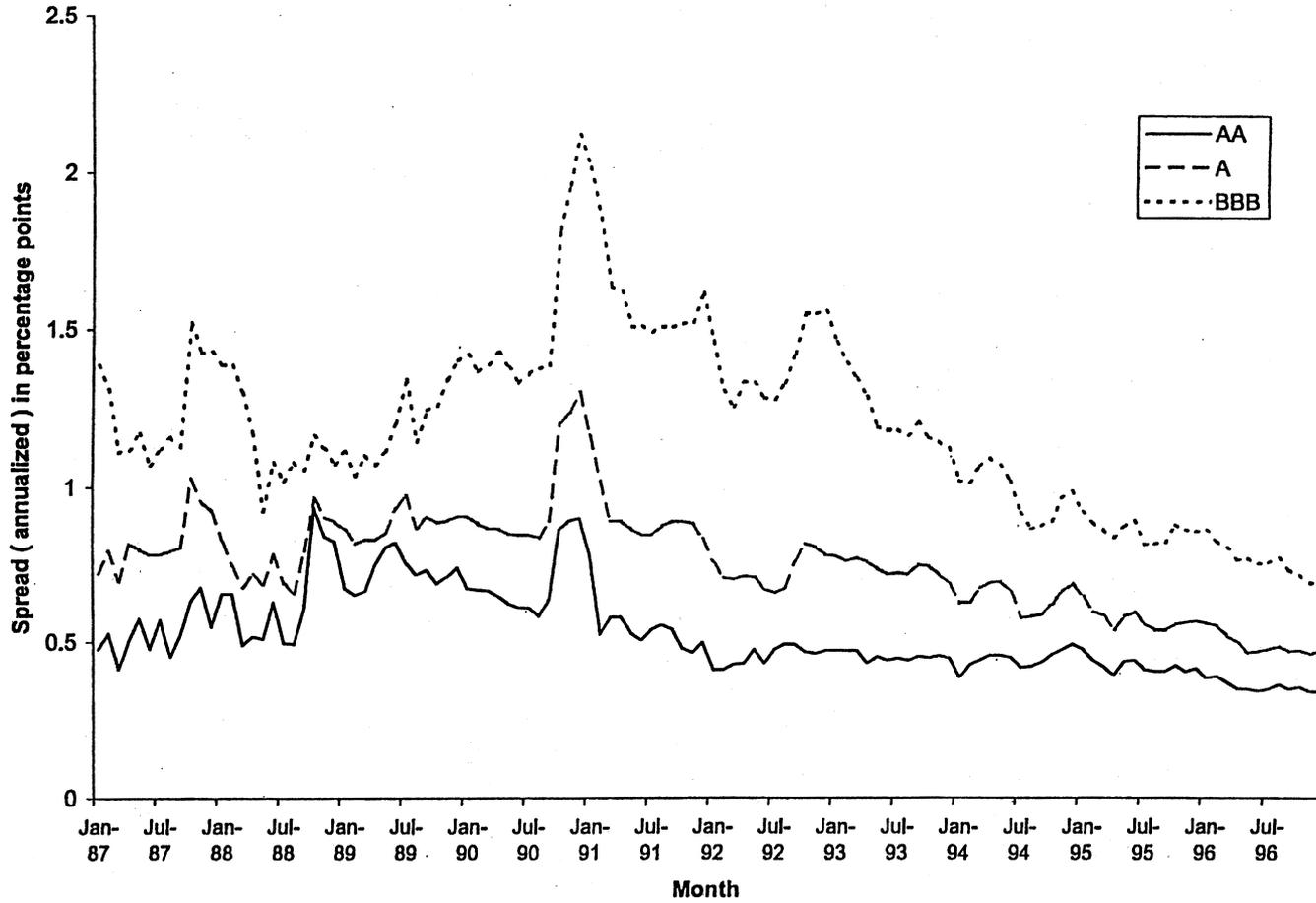


Figure 1. Empirical spreads on industrial bonds of six years maturity.

**Table II**  
**Average Root Mean Squared Errors**

This table contains the average root mean squared error of the difference between theoretical prices computed from the spot rates derived from the Gauss–Newton procedure and the actual bond invoice prices. Root mean squared error is measured in cents per \$100. For a given class of securities, the root mean squared error is calculated once per period. The number reported is the average of all the root mean squared errors within a class over the period indicated.

Period	Treasuries	Financial Sector			Industrial Sector		
		AA	A	BBB	AA	A	BBB
1987–1996	0.210	0.512	0.861	1.175	0.728	0.874	1.516
1987–1991	0.185	0.514	0.996	1.243	0.728	0.948	1.480
1992–1996	0.234	0.510	0.726	1.108	0.727	0.800	1.552

rating classes is very close to zero (less than one cent on a \$100 bond). Root mean squared error is a measure of the variance of errors within each rating class. The average root mean squared error between actual price and estimated price is shown in Table II. The average root mean square error of 21 cents per \$100 for Treasuries is comparable to the average root mean squared error found in other studies. Elton and Green (1998) had showed average absolute errors of about 16 cents per \$100 using GovPX data over the period June 1991 to September 1995. GovPX data are trade prices, yet the difference in error between the studies is quite small. Green and Odegaard (1997) used the Cox, Ingersoll, and Ross (1985) procedure to estimate spot rates using data from CRSP. Although their procedure and time period are different from ours, their errors again are about the same as those we find for government bonds in our data set (our errors are smaller). The data set and procedures we are using seem to produce errors in pricing government bonds comparable in size to those found by other authors.

The average root mean squared pricing errors become larger as we examine lower grades of bonds while the average error does not change. Average root mean squared pricing errors are over twice as large for AA's as for Treasuries. The root mean squared pricing errors for BBBs are almost twice those of AAs, with the errors in As falling in between. Thus, default risk leads not only to higher spot rates, but also to greater uncertainty as to the appropriate value of the bond. This is reflected in a higher root mean squared error (variance of pricing errors). This is an added source of risk and may well be reflected in higher risk premiums, a subject we investigate shortly.<sup>15</sup>

<sup>15</sup> In a separate paper, we explore whether the difference in theoretical price and invoice price is random or related to bond characteristics. Bond characteristics do explain some of the differences but the characteristics and relationships do not change the results in this paper.

## II. Estimating the Default Premium

In this section, we will estimate the magnitude of the spread that would exist under risk neutrality with the tax differences between corporates and governments ignored. Later in Section II we will introduce tax differences and examine whether expected default premium and taxes together are sufficient to explain the observed spot spread.

If investors are risk neutral, then discounting the expected cash flows from a bond at the appropriate government spot rate would produce the same value as discounting promised payments at corporate spot rates. In Appendix B, employing this insight, we show that in a risk-neutral world, the difference between corporate and government forward rates is given by

$$e^{-(r_{t+1}^C - r_{t+1}^G)} = (1 - P_{t+1}) + \frac{aP_{t+1}}{V_{t+1T} + C}, \quad (1)$$

where  $C$  is the coupon rate;  $P_{t+1}$  is the probability of bankruptcy in period  $t + 1$  conditional on no bankruptcy in an earlier period (the marginal default probabilities);  $a$  is the recovery rate assumed constant in each period;  $r_{t+1}^C$  is the forward rate as of time 0 from  $t$  to  $t + 1$  for corporate bonds;  $r_{t+1}^G$  is the forward rate as of time 0 from  $t$  to  $t + 1$  for government (risk-free) bonds; and,  $V_{t+1T}$  is the value of a  $T$  period bond at time  $t + 1$  given that it has not gone bankrupt in an earlier period.

Equation (1) can be used to directly estimate the spot rate spread that would exist in a risk-neutral world between corporate and government bonds for any risk class and maturity. To perform this estimation, one needs estimates of coupons, recovery rates, and marginal default probabilities. First, the coupon was set so that a 10-year bond with that coupon would be selling close to par in all periods.<sup>16</sup> The only estimates available for recovery rates by rating class are computed as a function of the rating at time of issuance. Table III shows these recovery rates.<sup>17</sup> Estimating marginal default probabilities is more complex. Marginal default probabilities are developed from a transition matrix employing the assumption that the transition process is stationary and Markovian. We employed two separate estimates of the transition matrix, one estimated by S&P (see Altman (1997)) and one estimated by Moody's (Carty and Fons (1994)).<sup>18</sup> These are the two principal rating agencies for corporate debt. The transition matrixes are shown in Table IV.

<sup>16</sup> We examined alternative reasonable estimates for coupon rates and found only second-order effects in our results. Although this might seem inconsistent with equation (1), note that from the recursive application of equation (1) changes in  $C$  are largely offset by opposite changes in  $V$ .

<sup>17</sup> Recovery rates available in the literature assume that these rates are independent of the age of a bond.

<sup>18</sup> Each row of the transition matrix shows the probability of having a given rating in one year contingent on starting with the rating specified by the row.

**Table III**  
**Recovery Rates\***

This table shows the percentage of par that a bond is worth one month after bankruptcy, given the rating shown in the first column.

Original Rating	Recovery Rate (%)
AAA	68.34
AA	59.59
A	60.63
BBB	49.42
BB	39.05
B	37.54
CCC	38.02
Default	0

\*From Altman and Kishore (1998).

In year one, the marginal probability of default can be determined directly from the transition matrix and default vector, and is, for each rating class, the proportion of defaults in year one. To obtain year two defaults, we first use the transition matrix to calculate the ratings going into year two for any bond starting with a particular rating in year one. Year two defaults are then the proportion in each rating class times the probability that a bond in that class defaults by year end.<sup>19</sup> Table V shows the marginal default probabilities by age and initial rating class determined from the Moody's and S&P transition matrixes. The entries in this table represent the probability of default in year  $t$  given an initial rating in year 0 and given that the bond was not in default in year  $t - 1$ .

The marginal probability of default increases for the high-rated debt and decreases for the low-rated debt. This occurs because bonds change rating classes over time.<sup>20</sup> For example, a bond rated AAA by S&P has zero probability of defaulting one year later. However, given that it has not previously defaulted, the probability of it defaulting 20 years later is 0.206 percent. In the intervening years, some of the bonds originally rated AAA have migrated to lower-rated categories where there is some probability of default. At the other extreme, a bond originally rated CCC has a probability of defaulting equal to 22.052 percent in the next year, but if it survives 19 years the probability of default in the next year is only 2.928 percent. If it survives 19 years, the bond is likely to have a higher rating. Despite this drift, bonds that were rated very highly at time 0 tend to have a higher probability of staying out of default 20 years later than do bonds that initially had a low

<sup>19</sup> Technically, it is the last column of the squared transition matrix divided by one minus the probability of default in period 1.

<sup>20</sup> These default probabilities as a function of years survived are high relative to prior studies, for example, Altman (1997) and Moody's (1998).

**Table IV**  
**One One-Year Transition Probability Matrix**

Panel A is taken from Carty and Fons (1994) and Panel B is from Standard and Poor's (1995). However, the category in the original references titled Non-Rated (which is primarily bonds that are bought back or issued by companies that merge) has been allocated to the other rating classes so that each row sums to one. Each entry in a row shows the probability that a bond with a rating shown in the first column ends up one year later in the category shown in the column headings.

Panel A: Moody's								
	Aaa (%)	Aa (%)	A (%)	Baa (%)	Ba (%)	B (%)	Caa (%)	Default (%)
Aaa	91.897	7.385	0.718	0.000	0.000	0.000	0.000	0.000
Aa	1.131	91.264	7.091	0.308	0.206	0.000	0.000	0.000
A	0.102	2.561	91.189	5.328	0.615	0.205	0.000	0.000
Baa	0.000	0.206	5.361	87.938	5.464	0.825	0.103	0.103
Ba	0.000	0.106	0.425	4.995	85.122	7.333	0.425	1.594
B	0.000	0.109	0.109	0.543	5.972	82.193	2.172	8.903
Caa	0.000	0.437	0.437	0.873	2.511	5.895	67.795	22.052
Default	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000

Panel B: Standard and Poor's								
	AAA (%)	AA (%)	A (%)	BBB (%)	BB (%)	B (%)	CCC (%)	Default (%)
AAA	90.788	8.291	0.716	0.102	0.102	0.000	0.000	0.000
AA	0.103	91.219	7.851	0.620	0.103	0.103	0.000	0.000
A	0.924	2.361	90.041	5.441	0.719	0.308	0.103	0.103
BBB	0.000	0.318	5.938	86.947	5.302	1.166	0.117	0.212
BB	0.000	0.110	0.659	7.692	80.549	8.791	0.989	1.209
B	0.000	0.114	0.227	0.454	6.470	82.747	4.086	5.902
CCC	0.228	0.000	0.228	1.251	2.275	12.856	60.637	22.526
Default	0.000	0.000	0.000	0.000	0.000	0.000	0.000	100.000

rating. However, rating migration means this does not hold for all rating classes. For example, note that after 12 years the conditional probability of default for CCCs is lower than the default probability for Bs. Why? Examining Table III shows that the odds of being upgraded to investment grade conditional on not defaulting is higher for CCC than B. Eventually, bonds that start out as CCC and continue to exist will be rated higher than those that start out as Bs. In short, the small percentage of CCC bonds that continue to exist for many years end up at higher ratings on average than the larger percentage of B bonds that continue to exist for many years.

Employing equation (1) along with the conditional default probabilities from Table V, the recovery rates from Table III, and the coupon rates estimated as explained earlier allows us to calculate the forward rates assuming risk neutrality and zero taxes. This is then converted to an estimate of the spot spread due to expected default under the same assumptions.

**Table V**  
**Evolution of Default Probability**

Probability of default in year  $n$  conditional on (a) a particular starting rating and (b) not having defaulted prior to year  $n$ . These are determined using the transition matrix shown in Table IV. Panel A is based on Moody's transition matrix of Table IV, Panel A, and Panel B is based on Standard and Poor's transition matrix of Table IV, Panel B.

Panel A: Moody's							
Year	Aaa (%)	Aa (%)	A (%)	Baa (%)	Ba (%)	B (%)	Caa (%)
1	0.000	0.000	0.000	0.103	1.594	8.903	22.052
2	0.000	0.004	0.034	0.274	2.143	8.664	19.906
3	0.001	0.011	0.074	0.441	2.548	8.355	17.683
4	0.002	0.022	0.121	0.598	2.842	8.003	15.489
5	0.004	0.036	0.172	0.743	3.051	7.628	13.421
6	0.008	0.053	0.225	0.874	3.193	7.246	11.554
7	0.013	0.073	0.280	0.991	3.283	6.867	9.927
8	0.019	0.095	0.336	1.095	3.331	6.498	8.553
9	0.027	0.120	0.391	1.185	3.348	6.145	7.416
10	0.036	0.146	0.445	1.264	3.340	5.810	6.491
11	0.047	0.174	0.499	1.331	3.312	5.496	5.743
12	0.060	0.204	0.550	1.387	3.271	5.203	5.141
13	0.074	0.234	0.599	1.435	3.218	4.930	4.654
14	0.089	0.265	0.646	1.474	3.157	4.678	4.258
15	0.106	0.297	0.691	1.506	3.092	4.444	3.932
16	0.124	0.329	0.733	1.532	3.022	4.229	3.662
17	0.143	0.362	0.773	1.552	2.951	4.030	3.435
18	0.163	0.394	0.810	1.567	2.878	3.846	3.241
19	0.184	0.426	0.845	1.578	2.806	3.676	3.074
20	0.206	0.457	0.877	1.585	2.735	3.519	2.928

Panel B: Standard and Poor's							
Year	AAA (%)	AA (%)	A (%)	BBB (%)	BB (%)	B (%)	CCC (%)
1	0.000	0.000	0.103	0.212	1.209	5.902	22.526
2	0.002	0.017	0.154	0.350	1.754	6.253	18.649
3	0.007	0.037	0.204	0.493	2.147	6.318	15.171
4	0.013	0.061	0.254	0.632	2.424	6.220	12.285
5	0.022	0.087	0.305	0.761	2.612	6.031	10.031
6	0.032	0.115	0.355	0.879	2.733	5.795	8.339
7	0.045	0.145	0.406	0.983	2.804	5.540	7.095
8	0.059	0.177	0.457	1.075	2.836	5.280	6.182
9	0.075	0.210	0.506	1.153	2.840	5.025	5.506
10	0.093	0.243	0.554	1.221	2.822	4.780	4.993
11	0.112	0.278	0.600	1.277	2.790	4.548	4.594
12	0.132	0.313	0.644	1.325	2.746	4.330	4.272
13	0.154	0.348	0.686	1.363	2.695	4.125	4.006
14	0.176	0.383	0.726	1.395	2.639	3.934	3.780
15	0.200	0.419	0.763	1.419	2.581	3.756	3.583
16	0.225	0.453	0.797	1.439	2.520	3.591	3.408
17	0.250	0.488	0.830	1.453	2.460	3.436	3.252
18	0.276	0.521	0.860	1.464	2.400	3.292	3.109
19	0.302	0.554	0.888	1.471	2.341	3.158	2.979
20	0.329	0.586	0.913	1.475	2.284	3.033	2.860

Table VI

**Mean, Minimum, and Maximum Spreads Assuming Risk Neutrality**

This table shows the spread of corporate spot rates over government spot rates when taxes are assumed to be zero, and default rates and recovery rates are taken into account. The corporate forward rates are computed using equation (6). These forward rates are converted to spot rates, which are then used to compute the spreads below.

Years	AA (%)	A (%)	BBB (%)
Panel A: Mean Spreads			
1	0.000	0.043	0.110
2	0.004	0.053	0.145
3	0.008	0.063	0.181
4	0.012	0.074	0.217
5	0.017	0.084	0.252
6	0.023	0.095	0.286
7	0.028	0.106	0.319
8	0.034	0.117	0.351
9	0.041	0.128	0.380
10	0.048	0.140	0.409
Panel B: Minimum Spreads			
1	0.000	0.038	0.101
2	0.003	0.046	0.132
3	0.007	0.055	0.164
4	0.011	0.063	0.197
5	0.015	0.073	0.229
6	0.020	0.083	0.262
7	0.025	0.093	0.294
8	0.031	0.104	0.326
9	0.038	0.116	0.356
10	0.044	0.128	0.385
Panel C: Maximum Spreads			
1	0.000	0.047	0.118
2	0.004	0.059	0.156
3	0.009	0.071	0.196
4	0.014	0.083	0.235
5	0.019	0.094	0.273
6	0.025	0.106	0.309
7	0.031	0.117	0.342
8	0.038	0.129	0.374
9	0.044	0.140	0.403
10	0.051	0.151	0.431

Table VI shows the zero spread due to expected default under risk-neutral valuation. The first characteristic to note is the size of the tax-free spread due to expected default relative to the empirical corporate spread discussed earlier. Our major conclusion of this section is that the zero tax spread from expected default is very small and does not account for much of the corporate spread. This can be seen numerically by comparing Tables I and VI and is illustrated graphically in Figure 2 for A-rated industrial bonds. One factor

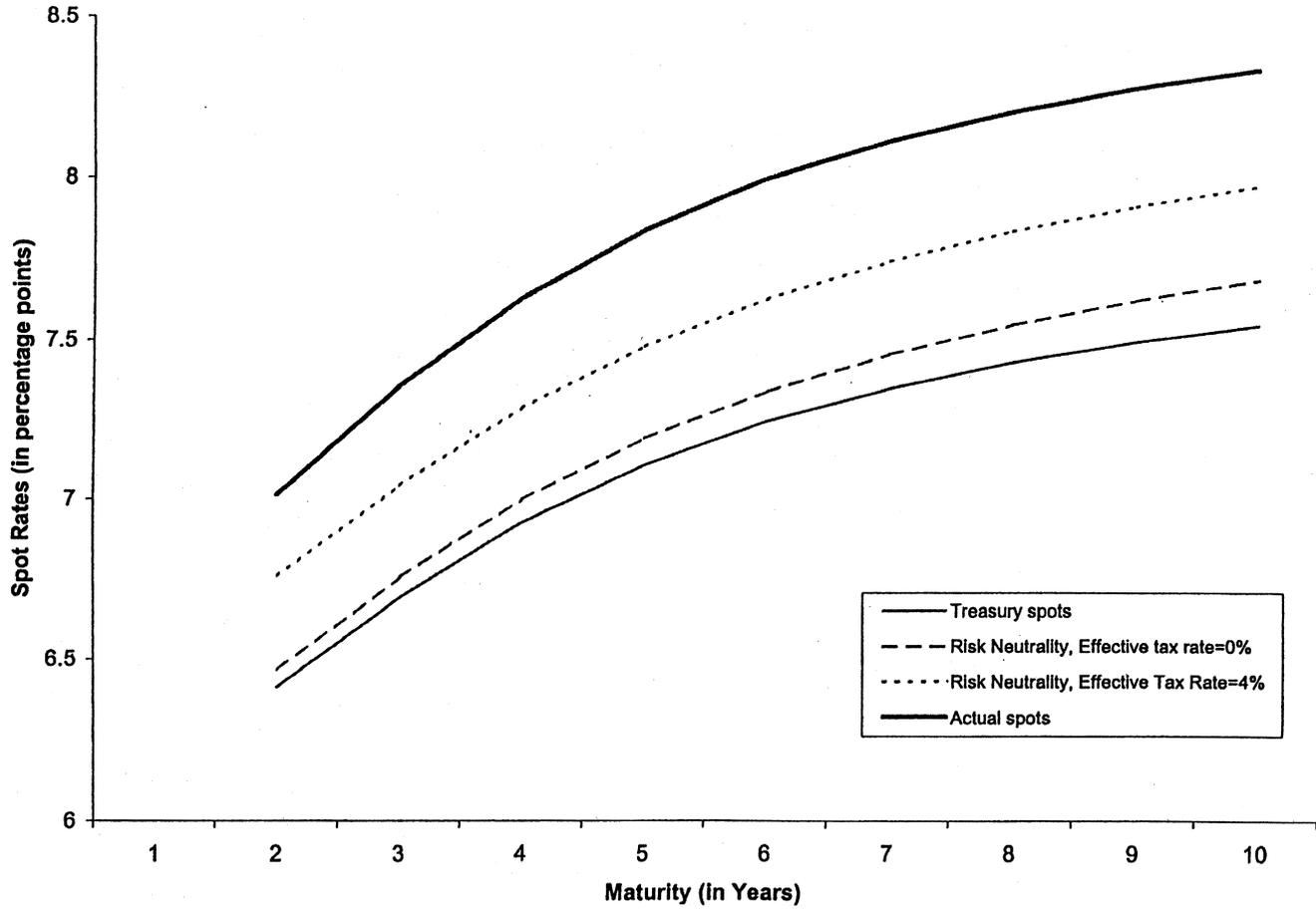


Figure 2. Spot rates for A rated industrial bonds and for treasuries.

that could cause us to underestimate the spread due to expected default is that our transition matrix estimates are not calculated over exactly the same period for which we estimate the spreads. However, there are three factors that make us believe that we have not underestimated default spreads. First, our default estimates shown in Table V are higher than those estimated in other studies. Second, the average default probabilities over the period where the transition matrix is estimated by Moody's and S&P are close to the average default probabilities in the period we estimate spreads (albeit default probabilities in the latter period are somewhat higher). Third, the S&P transition matrix that was estimated in a period with higher average default probability and that more closely matches the years in which we estimate spread results in lower estimates of defaults. However, as a further check on the effect of default rates on spreads, we calculated the standard deviation of year-to-year default rates over the 20 years ending 1996. We then increased the mean default rate by two standard deviations. This resulted in a maximum increase in spread in AA's of 0.004 percent and 0.023 percent for BBB's. Thus, even with extreme default rates, premiums due to expected losses are too small to account for the observed spreads. It also suggests that changes in premiums due to expected loss over time are too small to account for any significant part of the change in spreads over time.<sup>21</sup>

Also note from Table VI the zero tax spread due to default loss of AAs relative to BBBs. Although the spread for BBBs is higher, the difference in spreads because of differences in default experience is much less than the differences in the empirical corporate spreads. Differences in default rates cannot explain the differences in spreads between bonds of various rating classes. This strongly suggests that differences in spreads must be explained by other influences, such as taxes or risk premiums. The second characteristic of spreads due to expected default loss to note is the pattern of spreads as the maturity of the spot rate increases. The spread increases for longer maturity spots. This is the same pattern we observe for the empirical spreads shown in Table I. However, for AA and A the increase in premiums due to expected default loss with maturity is less than the increase in the empirical corporate spread.

### **III. Estimating The State Tax Premiums**

Another difference between government bonds and corporate bonds is that the interest payments on corporate bonds are subject to state tax with maximum marginal rates generally between 5 and 10 percent.<sup>22</sup> Because state

<sup>21</sup> Default rates are not separately reported for industrials and financials. Thus we cannot separately calculate the size of the spread needed for default. However, recognizing that differential default rates have little impact on the spread shows that differences in the default rates for the two classes of bonds are unimportant in explaining spread differences.

<sup>22</sup> For a very few cities such as New York, interest income is taxable at the city level. Companies have wide latitude in determining where this interest is earned. Thus, they have the ability, in particular, to avoid taxation. Thus, the tax burden is almost exclusively at the state level and we will refer to it in this way.

tax is deductible from income for the purpose of federal tax, the burden of state tax is reduced by the federal tax rate. Nevertheless, state taxes could be a major contributor to the spreads. For example, if the coupon was 10 percent and effective state taxes were 5 percent, state taxes alone would result in a  $\frac{1}{2}$  percent spread ( $0.05 \times 0.10$ ). To analyze the impact of state taxes on spreads, we introduced taxes into the analysis developed in the prior section. The derivation is contained in Appendix C. The final equation that parallels equation (1) is

$$e^{-(r_{it+1}^C - r_{it+1}^G)} = (1 - P_{t+1}) + \frac{aP_{t+1}}{C + V_{t+1T}} - \frac{[C(1 - P_{t+1}) - (1 - a)P_{t+1}]}{C + V_{t+1T}} t_s(1 - t_g), \quad (2)$$

where  $t_s$  is the state tax rate;  $t_g$  is the federal tax rate, and other terms are as before.

The first two terms on the right-hand side are identical to the terms shown before when only default risk was taken into account. The last term is the new term that captures the effect of taxes. Taxes enter in two ways. First, the coupon is taxable and its value is reduced by taxes and is paid with probability  $(1 - P_{t+1})$ . Second, if the firm defaults (with probability  $P_{t+1}$ ), the amount lost in default is a capital loss and taxes are recovered. Note that because state taxes are a deduction against federal taxes, the marginal impact of state taxes is  $t_s(1 - t_g)$ . Equation (2) is used to estimate the forward rate spread caused by the combined effects of loss due to expected default and taxes. Estimation of the forward rate spread requires, in addition to the data employed in the previous section of this paper, estimates of the term  $t_s(1 - t_g)$  which we subsequently refer to as  $\tau$ .

There is no direct way to measure the size of the tax terms. We employed three different procedures to measure the size of  $\tau$ . The first, and the one we prefer, involves a grid search. We examine 11 different values of tax rates ranging from 0 percent to 10 percent in steps of 1 percent. For each tax rate we estimate the after-tax cash flow for each bond in every month in our sample. This was done using cash flows as defined in the multiperiod version of equation (C1) in Appendix C. Then for each month, rating class, and tax rate, we estimate the spot rates using the Nelson–Siegel procedure discussed in Appendix A, but now applied to after-tax expected cash flows. These spot yield curves are then applied to the appropriate after-tax expected cash flows to prices of all bonds in each rating class in each month. The difference between this computed price and the actual price is calculated for each tax rate. The tax rate that resulted in the smallest mean squared error between calculated price and actual price is determined, and we find that an effective tax rate of four percent results in the smallest mean squared pricing error. In addition, the four percent rate produces errors

that were lower (at the five percent significance level) than any other rate except three percent. Because errors were lower on average with the four percent rate, we employ this rate for later analysis.<sup>23</sup>

As a reality check on the estimation of  $\tau$ , we examined the tax codes in existence in each state. For most states, maximum marginal state tax rates range between 5 percent and 10 percent.<sup>24</sup> Because the marginal tax rate used to price bonds should be a weighted average of the active traders, we assume that a maximum marginal tax rate would be approximately the midpoint of the range of maximum state taxes, or 7.5 percent. In almost all states, state tax for financial institutions (the main holder of bonds) is paid on income subject to federal tax. Thus, if interest is subject to maximum state rates, it must also be subject to maximum federal tax, and we assume the maximum federal tax rate of 35 percent. This yields an estimate of  $\tau$  of 4.875 percent.

A definite upper limit on the size of  $\tau$  can be established by examining AA bonds (our highest rated category) and assuming that no risk premium exists for these bonds. If we make this assumption, the derived tax rate that explains AA spreads is 6.7 percent. There are many combinations of federal and state taxes that are consistent with this number. However, as noted above, because state tax is paid on federal income, it is illogical to assume a high state rate without a corresponding high federal rate. Thus, the only pair of rates that would explain spreads on AAs is a state tax rate of 10.3 percent and a federal rate of 35 percent. There are very few states with a 10 percent rate. Thus, it is hard to explain spreads on AA bonds with taxes and default rates. A risk premium appears to be present even for these bonds.

The corporate spreads that arise from the combined effects of expected default loss and our three tax estimates are shown in Table VII. In Table VII we have used the forward rates determined from equation (2) to calculate spot rates. Note first that the spreads in Table VII are less than those found empirically, as shown in Table I, and that, for our best estimate of effective state taxes (four percent) or for the estimate obtained from estimating rates directly, state taxes are more important than expected loss due to default in explaining spreads. This can be seen by comparing Tables VII, Panels A and B, and Table VI, or by examining Figure 2. Recall that increasing default probabilities by two standard deviations only increased the spread for AA bonds by 0.003 percent. Thus, increasing defaults to an extreme historical level plus adding on maximum or estimated tax rates are insufficient to explain the corporate spreads found empirically.

Examining Panel C of Table VII shows the spread when we apply the effective tax rate of 6.7 percent that explains AA spread to A and BBB rated bonds. Note that the tax rate that explains the spreads on AA debt underestimate the spreads on A and BBB bonds. Taxes, expected default losses,

<sup>23</sup> One other estimate in the literature that we are aware of is that produced by Severn and Stewart (1992), who estimate state taxes at five percent.

<sup>24</sup> See Commerce Clearing House (1997).

**Table VII**  
**Mean, Minimum, and Maximum Spreads with Taxes,**  
**Assuming Risk Neutrality**

This table shows the spread of corporate spot rates over government spot rates when taxes as well as default rates and recovery rates are taken into account. The corporate forward rates are computed using equation (9). These forward rates are converted to spot rates, which are then used to compute the spreads below.

Years	AA (%)	A (%)	BBB (%)
Panel A: Mean Spreads with Effective Tax Rate of 4.875%			
1	0.358	0.399	0.467
2	0.362	0.410	0.501
3	0.366	0.419	0.535
4	0.370	0.429	0.568
5	0.375	0.438	0.601
6	0.379	0.448	0.632
7	0.383	0.457	0.662
8	0.388	0.466	0.691
9	0.393	0.476	0.718
10	0.398	0.486	0.744
Panel B: Mean Spreads with Effective Tax Rate of 4.0%			
1	0.292	0.334	0.402
2	0.296	0.344	0.436
3	0.301	0.354	0.470
4	0.305	0.364	0.504
5	0.309	0.374	0.537
6	0.314	0.383	0.569
7	0.319	0.393	0.600
8	0.324	0.403	0.629
9	0.329	0.413	0.657
10	0.335	0.423	0.683
Panel C: Mean Spreads with Effective Tax Rate of 6.7%			
1	0.496	0.537	0.606
2	0.501	0.547	0.639
3	0.505	0.557	0.672
4	0.508	0.566	0.704
5	0.512	0.575	0.735
6	0.516	0.583	0.765
7	0.520	0.592	0.794
8	0.524	0.600	0.821
9	0.528	0.609	0.847
10	0.532	0.618	0.871

and the risk premium inherent in AA bonds underestimate the corporate spread on lower-rated bonds. Furthermore, as shown in Table VII, Panel C, the amount of the underestimate goes up as the quality of the bonds examined goes down. The inability of tax and expected default losses to explain

the corporate spread for AA's even at extreme tax rates and the inability to explain the difference in spreads between AA's and BBB's suggest a nonzero risk premium. State taxes have been ignored in almost all modeling of the spread (see, e.g., Das and Tufano (1996), Jarrow et al. (1997), and Duffee (1998)). Our results indicate that state taxes should be an important influence that should be included in such models if they are to help us understand the causes of corporate bond spreads.

#### **IV. Risk Premiums For Systematic Risk**

As shown in the last section, premiums due to expected default losses and state tax are insufficient to explain the corporate bond spread. Thus, we need to examine the unexplained spread to see if it is indeed a risk premium. There are two issues that need to be addressed. What causes a risk premium and, given the small size of the expected default loss, why is the risk premium so large?<sup>25</sup>

If corporate bond returns move systematically with other assets in the market whereas government bonds do not, then corporate bond expected returns would require a risk premium to compensate for the nondiversifiability of corporate bond risk, just like any other asset. The literature of financial economics provides evidence that government bond returns are not sensitive to the influences driving stock returns.<sup>26</sup> There are two reasons why changes in corporate spreads might be systematic. First, if expected default loss were to move with equity prices, so while stock prices rise default risk goes down and as stock prices fall default risk goes up, it would introduce a systematic factor. Second, the compensation for risk required in capital markets changes over time. If changes in the required compensation for risk affects both corporate bond and stock markets, then this would introduce a systematic influence. We believe the second reason to be the dominant influence. We shall now demonstrate that such a relationship exists and that it explains most of the spread not explained by expected default losses and taxes. We demonstrate this by relating unexplained spreads (corporate spreads less both the premium for expected default and the tax premium as determined from equation (2)) to variables that have been used as systematic risk factors in the pricing of common stocks. By studying sensitivity to these risk factors, we can estimate the size of the premium required

<sup>25</sup> An alternative possibility to that discussed shortly is that we might expect a large risk premium despite the low probability of default for the following reasons. Bankruptcies tend to cluster in time and institutions are highly levered, so that even with low average bankruptcy losses, there is still a significant chance of financial difficulty at an uncertain time in the future and thus there is a premium to compensate for this risk. In addition, even if the institutional bankruptcy risk is small, the consequences of the bankruptcy of an individual issue on a manager's career may be so significant as to induce decision makers to require a substantial premium.

<sup>26</sup> See, for example, Elton (1999).

and see if it explains the remaining part of the spread. After examining the importance of systematic risk, we shall examine whether incorporating expected defaults as a systematic factor improves our ability to explain spreads.<sup>27</sup>

To examine the impact of sensitivities on unexplained spreads we need to specify a return-generating model. We can write a general return-generating model as

$$R_t = a + \sum_j \beta_j f_{jt} + e_t \quad (3)$$

for each year (2–10) and each rating class, where  $R_t$  is the return during month  $t$ ;  $\beta_j$  is the sensitivity of changes in the spread to factor  $j$ ; and  $f_{jt}$  is the return on factor  $j$  during month  $t$ . The factors are each formulated as the difference in return between two portfolios (zero net investment portfolios).

As we show below, changes in the spread have a direct mathematical relationship with the difference in return between a corporate bond and a government bond. The relationship between the return on a constant maturity portfolio and the spread in spot rates is easy to derive. Thus, if either changes in spreads or the difference in returns between corporate bonds and government bonds are related to a set of factors (systematic influences), then the other must also be related to the same factors.

Let  $r_{t,m}^c$  and  $r_{t,m}^G$  be the spot rates on corporate and government bonds that mature  $m$  periods later, respectively. Then the price of a pure discount bond with face value equal to one dollar is

$$P_{t,m}^c = e^{-r_{t,m}^c \cdot m} \quad (4)$$

and

$$P_{t,m}^G = e^{-r_{t,m}^G \cdot m}, \quad (5)$$

and one month later the price of  $m$  period corporate and government bonds are

$$P_{t+1,m}^c = e^{-r_{t+1,m}^c \cdot m} \quad (6)$$

and

$$P_{t+1,m}^G = e^{-r_{t+1,m}^G \cdot m}. \quad (7)$$

<sup>27</sup> Throughout this section we will assume a four percent effective state tax rate, which is our estimate from the prior section.

Thus, the part of the return on a constant maturity  $m$  period zero-coupon bond from  $t$  to  $t + 1$  due to a change in the  $m$  period spot rate is<sup>28</sup>

$$R_{t,t+1}^c = \ln \frac{e^{-r_{t+1,m}^c \cdot m}}{e^{-r_{t,m}^c \cdot m}} = m(r_{t,m}^c - r_{t+1,m}^c) \quad (8)$$

and

$$R_{t,t+1}^G = \ln \frac{e^{-r_{t+1,m}^G \cdot m}}{e^{-r_{t,m}^G \cdot m}} = m(r_{t,m}^G - r_{t+1,m}^G), \quad (9)$$

and the differential return between corporate and government bonds due to a change in spread is

$$R_{t,t+1}^c - R_{t,t+1}^G = -m[(r_{t+1,m}^c - r_{t+1,m}^G) - (r_{t,m}^c - r_{t,m}^G)] = -m\Delta S_{t,m}, \quad (10)$$

where  $\Delta S_{t,m}$  is the change in spread from time  $t$  to  $t + 1$  on an  $m$  period constant maturity bond. Thus, the difference in return between corporate and government bonds due solely to a change in spread is equal to minus  $m$  times the change in spread.

Recognize that we are interested in the unexplained spread that is the difference between the corporate government spread and that part of the spread that is explained by expected default loss and taxes. Adding a superscript to note that we are dealing with that part of the spread on corporate bonds that is not explained by expected default loss and taxes, we can write the unexplained differential in returns as

$$R_{t,t+1}^{uc} - R_{t,t+1}^G = -m[(r_{t+1,m}^{uc} - r_{t+1,m}^G) - (r_{t,m}^{uc} - r_{t,m}^G)] = -m\Delta S_{t,m}^u. \quad (11)$$

There are many forms of a multi-index model that we could employ to study unexplained spreads. We chose to concentrate our results on the Fama and French (1993) three-factor model because of its wide use in the literature, but we also investigated other models including the single-index model, and some of these results will be discussed in footnotes.<sup>29</sup> The Fama-French model employs the excess return on the market, the return on a portfolio of small stocks minus the return on a portfolio of large stocks (the SMB factor), and the return on a portfolio of high minus low book-to-market stocks (the HML factor) as its three factors.

<sup>28</sup> This is not the total return on holding a corporate or government bond, but rather the portion of the return due to changing spread (the term we wish to examine).

<sup>29</sup> We used two other multifactor models, the Connor and Korajczyk (1993) empirically derived model and the multifactor model tested by us earlier. See Elton et al. (1999). These results will be discussed in footnotes. We thank Bob Korajczyk for supplying us with the monthly returns on the Connor and Korajczyk factors.

Table VIII shows the results of regressing return of corporates over governments derived from the change in unexplained spread for industrial bonds (as in equation (5)) against the Fama–French factors.<sup>30</sup> The regression coefficient on the market factor is always positive and is statistically significant 20 out of 27 times. This is the sign we would expect on the basis of theory. This holds for the Fama–French market factor, and also holds (see Table VIII) for the other Fama–French factors representing size and book-to-market ratios. The return is positively related to the SMB factor and to the HML factor.<sup>31</sup> Notice that the sensitivity to all of these factors tends to increase as maturity increases and to increase as quality decreases. This is exactly what would be expected if we were indeed measuring risk factors. Examining financials shows similar results except that the statistical significance of the regression coefficients and the size of the  $R^2$  is higher for AA's.

It appears that the change in spread not related to taxes or expected default losses is at least in part explained by factors that have been successful in explaining changes in returns over time in the equity market. We will now turn to examining cross-sectional differences in average unexplained premiums. If there is a risk premium for sensitivity to stock market factors, differences in sensitivities should explain differences in the unexplained premium across corporate bonds of different maturity and different rating class. We have 27 unexplained spreads for industrial bonds and 27 for financial bonds since maturities range from 2 years through 10 years, and there are three rating classifications. When we regress the average unexplained spread against sensitivities for industrial bonds, the cross-sectional  $R^2$  adjusted for degrees of freedom is 0.32, and for financials it is 0.58. We have been able to account for almost one-third of the cross-sectional variation in unexplained premiums for industrials and one-half for financial bonds.<sup>32</sup>

Another way to examine this is to ask how much of the unexplained spread the sensitivities can account for. For each maturity and risk class of bonds, what is the size of the unexplained spread that existed versus the size of the estimated risk premium where the estimated premium is determined by multiplying the sensitivity of the bonds to each of the three factors times the price of each of these factors over the time period? For industrials, the average

<sup>30</sup> If we find no systematic influences it does not imply that the unexplained returns are not risk premiums due to systematic influences. It may simply mean that we have failed to uncover the correct systematic influences. However, finding a relationship is evidence that the unexplained returns are due to a risk premium.

<sup>31</sup> The results are almost identical using the Connor and Korajczyk empirically derived factors or the Elton et al. (1999) model. When a single-factor model is used, 20 out of 27 betas are significant with an of  $R^2$  about 0.10.

<sup>32</sup> Employing a single index model using sensitivity to the excess return on the S&P index leads to  $R^2$  of 0.21 and 0.43 for industrial and financial bonds, respectively. Because returns on government bonds are independent of stock factors, the beta of the change in spreads with stock excess returns is almost completely due to the effect of the stock market return on corporate bond returns. The beta for BBB industrials averages 0.26, whereas for five-year bonds, the betas ranged from 0.12 to 0.76 across rating categories. Although bond betas are smaller than stock betas, the premium to be explained is also much smaller.

**Table VIII**  
**Relationship Between Returns and Fama–French Risk Factors**

This table shows the results of the regression of returns due to a change in the unexplained spread on the Fama–French risk factors, viz. (a) the market excess return (over T-bills) factor, (b) the small minus big factor, and (c) the high minus low book-to-market factor. The results reported below are for industrial corporate bonds. Similar results were obtained for bonds of financial firms. The values in parentheses are  $t$ -values.

Maturity	Constant	Market	SMB	HML	Adj- $R^2$
Panel A: Industrial AA-rated Bonds					
2	-0.0046 (-0.297)	0.0773 (2.197)	0.1192 (2.318)	-0.0250 (-0.404)	0.0986
3	-0.0066 (-0.286)	0.1103 (2.114)	0.2045 (2.680)	0.0518 (0.563)	0.0858
4	-0.0058 (-0.210)	0.1238 (1.983)	0.2626 (2.877)	0.0994 (0.903)	0.0846
5	-0.0034 (-0.109)	0.1260 (1.791)	0.3032 (2.949)	0.1261 (1.018)	0.0801
6	-0.0001 (-0.003)	0.1222 (1.463)	0.3348 (2.742)	0.1414 (0.961)	0.0608
7	0.0035 (0.077)	0.1157 (1.116)	0.3621 (2.391)	0.1514 (0.829)	0.0374
8	0.0073 (0.129)	0.1080 (0.839)	0.3873 (2.059)	0.1586 (0.700)	0.0195
9	0.0112 (0.163)	0.0996 (0.635)	0.4119 (1.798)	0.1650 (0.598)	0.0076
10	0.0151 (0.184)	0.0912 (0.489)	0.4356 (1.598)	0.1704 (0.519)	-0.0002
Panel B: Industrial A-rated Bonds					
2	-0.0081 (-0.437)	0.1353 (3.202)	0.1831 (2.965)	0.0989 (1.329)	0.1372
3	-0.0119 (-0.534)	0.1847 (3.631)	0.3072 (4.134)	0.1803 (2.013)	0.2068
4	-0.0123 (-0.501)	0.2178 (3.904)	0.3911 (4.796)	0.2619 (2.666)	0.2493
5	-0.0105 (-0.403)	0.2419 (4.068)	0.4498 (5.176)	0.3424 (3.270)	0.2754
6	-0.0077 (-0.262)	0.2616 (3.899)	0.4952 (5.050)	0.4222 (3.573)	0.2647
7	-0.0044 (-0.125)	0.2792 (3.480)	0.5345 (4.560)	0.5014 (3.549)	0.226
8	-0.0009 (-0.020)	0.2958 (3.032)	0.5709 (4.003)	0.5805 (3.378)	0.1828
9	0.0028 (0.053)	0.3121 (2.654)	0.6059 (3.525)	0.6596 (3.185)	0.1469
10	0.0064 (0.105)	0.3282 (2.357)	0.6407 (3.149)	0.7385 (3.012)	0.1198
Panel C: Industrial BBB-rated Bonds					
2	0.0083 (0.276)	0.1112 (1.626)	0.3401 (3.403)	0.1259 (1.045)	0.0969
3	0.0094 (0.255)	0.1691 (2.010)	0.4656 (3.787)	0.2922 (1.972)	0.1263
4	0.0084 (0.209)	0.2379 (2.601)	0.5836 (4.365)	0.4605 (2.858)	0.1798
5	0.0062 (0.153)	0.3132 (3.406)	0.6987 (5.199)	0.6263 (3.867)	0.2585
6	0.0034 (0.080)	0.3919 (4.025)	0.8127 (5.711)	0.7901 (4.607)	0.3126
7	0.0004 (0.008)	0.4720 (4.147)	0.9260 (5.567)	0.9522 (4.750)	0.3122
8	-0.0028 (-0.045)	0.5528 (3.951)	1.0395 (5.084)	1.1139 (4.520)	0.2807
9	-0.006 (-0.079)	0.6341 (3.685)	1.1529 (4.585)	1.2754 (4.209)	0.2445
10	-0.0092 (-0.101)	0.7154 (3.446)	1.2662 (4.173)	1.4370 (3.930)	0.2136

risk premium is 0.813, whereas using the sensitivities and factor prices we would estimate it to be 0.660. For financials, the actual risk premium is 0.934, but using the estimated beta and prices, it is 0.605. In short, 85 percent of the industrial unexplained spread is accounted for by the three risk sensitivities and for financials it is 67 percent. If a single-factor model were used, the amount of the risk premium explained by the systematic risk would be reduced by more than one-third. Thus, the additional factors are important. Note that whether we use the cross-sectional explanatory power or the size of the estimate relative to the realized risk premium, we see that standard risk measures have been able to account for a high percentage of the unexplained spread.<sup>33</sup>

We tried one more set of tests. One possible explanation for our results is that the Fama–French factors are proxies for changes in default expectations. If this is the case, in cross section, the sensitivity of unexplained spreads to the factors may in part be picking up the market price of systematic changes in default expectations. To test this, we added several measures of changes in default risk to equation (3) as a fourth factor. We tried actual changes (perfect forecasting) and several distributed lag and lead models. None of the results were statistically significant or had consistent signs across different groups of bonds. Changes in default risk do not seem to contain any additional information about systematic risk beyond the information already captured by the Fama–French factors.

In this section we have shown that the change in unexplained spread is related to factors that are considered systematic in the stock market. Modern risk theory states that systematic risk needs to be compensated for and thus, common equity has to earn a risk premium. Changes in corporate spreads lead to changes in return on corporates and thus, returns on corporates are also systematically related to common stock factors with the same sign as common equity. If common equity receives a risk premium for this systematic risk, then corporate bonds must also earn a risk premium. We have shown that sensitivity to the factors that are used to explain risk premiums in common stocks explains between 2/3 and 85 percent of the spread in corporate and government rates that is not explained by the difference between promised and expected payments and taxes. This is strong evidence of the existence of a risk premium of a magnitude that has economic significance and provides an explanation as to why spreads on corporate bonds are so large.

## V. Conclusion

In this paper we have examined the difference between spot rates on corporate and government bonds. We have shown that the spread can almost entirely be explained by three influences: the loss from expected

<sup>33</sup> Duffie and Singleton (1997) relate swap spreads to a series of interest rate variables. They find that the largest effect on spreads is prior shocks in this spread and changes in the spread between different rated corporate bonds.

defaults, state and local taxes which must be paid on corporate bonds but not on government bonds, and a premium required for bearing systematic risk. We supply estimates of the magnitude of each of these influences.

Several findings are of particular interest. The ratings of corporate bonds, whether provided by Moody's or Standard and Poor's, provide material information about spot rates. However, only a small part of the spread between corporate and treasuries and the difference in spreads on bonds with different ratings is explained by the expected default loss. For example, for 10-year A-rated industrials, expected loss from default accounts for only 17.8 percent of the spread.

Differential taxes are a more important influence on spreads. Taxes account for a significantly larger portion of the differential between corporate and treasuries than do expected losses. For example, for 10-year A-rated bonds, taxes accounted for 36.1 percent of the difference compared to the 17.8 percent accounted for by expected loss. State and local taxes are important because they are paid on the entire coupon of corporate bonds, not just on the difference in coupon between corporate and treasuries. Despite the importance of the state and local taxes in explaining return differentials, their impact has been ignored in almost all prior studies of corporate rates.

Even after we account for the impact of default and taxes, there still remains a large part of the differential between corporate and treasuries that remains unexplained. In the case of 10-year corporates, 46.17 percent of the difference is unexplained by taxes or expected default. We have shown that the vast majority of this difference is compensation for systematic risk and is affected by the same influences that affect systematic risks in the stock market. Making use of the Fama–French factors, we show that as much as 85 percent of that part of the spread that is not accounted for by taxes and expected default can be explained as a reward for bearing systematic risk.

In summary, we have been able to account for almost all of the differences between corporate rates and government rates. We have provided explicit estimates of the size of the these influences and we have shown that both state taxes and risk premiums are more important than the literature of financial economics has suggested.

### **Appendix A.** **Determining Yield to Maturity on Zeros (Spot Rates)**

Although there are several methods of determining spot rates given a set of bond prices, because of its simplicity and proven success in deriving spots we have adopted the methodology put forth by Nelson and Siegel (1987).

The Nelson and Siegel methodology involves fitting the following equations to all bonds in a given risk category to obtain the spot rate that is appropriate at any point in time:

$$D_t = e^{-r_t t}, \quad (\text{A1})$$

and

$$r_t = a_0 + (a_1 + a_2) \left[ \frac{1 - e^{-a_3 t}}{a_3 t} \right] - a_2 e^{-a_3 t}, \quad (\text{A2})$$

where  $D_t$  is the present value as of time 0 for a payment that is received  $t$  periods in the future;  $r_t$  is the spot rate at time 0 for a payment to be received at time  $t$ ; and  $a_0$ ,  $a_1$ ,  $a_2$ , and  $a_3$  are parameters of the model.

The Nelson and Siegel procedure is used to estimate spot rates for different maturities for both Treasury bonds and for bonds within each corporate rating class for every month over the time period January 1987 through December 1996. The estimation procedure allows us, on any date, to use corporate coupon, principal payments, and prices of all bonds within the same rating class to estimate the full spot yield (discount rate) curve that best explains the prices of all bonds in that rating class on that date.<sup>34</sup>

### **Appendix B. Measuring the Default Premium in a Risk-Neutral World Without State Taxes**

If investors were risk neutral (risk neutrality), the expected cash flows could be discounted at the government bond rate to obtain the value of a corporate bond. Consider a two-period bond using expected cash flows and risk neutrality. For simplicity, assume its par value at maturity is \$1. We wish to determine its value at time 0 and we do so recursively by valuing it first at time 1 (as seen at time 0) and then at time 0. Its value as of time 1 when it is a one-period bond has three component parts: the value of the expected coupon to be received at period 2, the value of the expected principal to be received at period 2 if the bond goes bankrupt at period 2, and the value of the principal if the bond survives where all expectations are conditional on the bond surviving to period 1. For a bond with a face value of \$1 this can be expressed as<sup>35</sup>

$$V_{12} = [C(1 - P_2) + aP_2 + (1 - P_2)]e^{-r_{12}^G}, \quad (\text{B1})$$

<sup>34</sup> We also used the McCulloch procedure and found that numerical results were similar and all of the conclusions of this paper were unchanged.

<sup>35</sup> The assumption of receiving a constant proportion of face value has been made in the literature by Duffie (1998). We are assuming that default payment occurs at the time of default. This is consistent with the evidence that default occurs because of an inability to meet a payment. We also assume that recovery rate is a percentage of par. This is how all data is collected (e.g., Altman (1997)).

where  $C$  is the coupon rate;  $P_t$  is the probability of bankruptcy in period  $t$  conditional on no bankruptcy in an earlier period;  $a$  is the recovery rate assumed constant in each period;  $r_{tt+1}^G$  is the forward rate as of time 0 from  $t$  to  $t + 1$  for government (risk-free) bonds;<sup>36</sup> and  $V_{tT}$  is the value of a  $T$  period bond at time  $t$  given that it has not gone bankrupt in an earlier period. Alternatively, we can value the bond using promised cash flows, according to

$$V_{12} = (C + 1)e^{-r_{12}^C}, \tag{B2}$$

where  $r_{tt+1}^C$  is the forward rate from  $t$  to  $t + 1$  for corporate bonds.

Equating the two values and rearranging to solve for the difference between corporate and government forward rates, we have

$$e^{-(r_{12}^C - r_{12}^G)} = (1 - P_2) + \frac{aP_2}{(1 + C)}. \tag{B3}$$

At time 0, the value of the two-period bond using risk neutral valuation is

$$V_{02} = [C(1 - P_1) + aP_1 + (1 - P_1)V_{12}]e^{-r_{01}^G} \tag{B4}$$

and using promised cash flows, its value is

$$V_{02} = [C + V_{12}]e^{-r_{01}^C}. \tag{B5}$$

Equating these expressions for  $V_{02}$  and solving for the difference in one-period spot (or forward) rates, we have

$$e^{-(r_{01}^C - r_{01}^G)} = (1 - P_1) + \frac{aP_1}{V_{12} + C}. \tag{B6}$$

In general, in period  $t$  the difference in forward rates is<sup>37</sup>

$$e^{-(r_{tt+1}^C - r_{tt+1}^G)} = (1 - P_{t+1}) + \frac{aP_{t+1}}{V_{t+1T} + C}, \tag{B7}$$

where  $V_{TT} = 1$ .

<sup>36</sup> We discount at the forward rate because this is the rate which can be contracted upon at time 0 for moving money across time.

<sup>37</sup> The difference in forward rates may vary across bonds with different coupons, even for bonds of the same rating class because, as discussed earlier, arbitrage on promised payments is an approximation that holds exactly only under certain assumptions (see Duffie and Singleton (1999)). If these assumptions do not hold, the estimates of spot rates obtained empirically are averages across bonds with different coupons and one single spot rate would not hold exactly for all bonds. Nevertheless, even in this case, given the size of the pricing error found in the previous section, assuming one rate is a good approximation.

### Appendix C. Estimating the Impact of State Taxes

To analyze the impact of state taxes on spreads, we introduced the taxes into the analysis developed in Section II. For a one-period bond maturing at \$1, the basic valuation equation after state taxes is

$$V_{01} = [C(1 - P_1)(1 - t_s(1 - t_g)) + aP_1 + (1 - a)P_1(t_s(1 - t_g)) + (1 - P_1)]e^{-r_{01}^G}, \quad (\text{C1})$$

where  $t_s$  is the state tax rate,  $t_g$  is the federal tax rate, and other terms are as before.

Equation (C1) has two terms that differ from those when taxes are not present. The change in the first term represents the payment of taxes on the coupon. The new third term is the tax refund due to a capital loss if the bond defaults.

The valuation on promised cash flows is

$$V_{01} = [C + 1]e^{-r_{01}^C}, \quad (\text{C2})$$

Equating the two expressions for  $V_{01}$  and solving for the difference between corporate and government rates, we have

$$e^{-(r_{01}^C - r_{01}^G)} = (1 - P_1) + \frac{aP_1}{1 + C} - \frac{[C(1 - P_1) - (1 - a)P_1]}{1 + C} (t_s)(1 - t_g). \quad (\text{C3})$$

As in Appendix B, these equations can be generalized to the  $T$  period case. The final equation is

$$(1 - P_{t+1}) + \frac{aP_{t+1}}{C + V_{t+1T}} - \frac{[C(1 - P_{t+1}) - (1 - a)P_{t+1}]}{C + V_{t+1T}} t_s(1 - t_g) = e^{-(r_{t+1}^C - r_{t+1}^G)}. \quad (\text{C4})$$

This equation is used to estimate the forward rate spread because of loss due to expected default and taxes.

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