Risk Management: Course Summary

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Goals of risk management:
- Understanding the risk profile of the entire portfolio for better risk/return positioning (typically, a large-scale problem)
- This requires identification and measurement of market, credit, and operational risk, as well as integrating risk

Risk tools:
- VAR is a measure of the “worst” loss over the horizon that will not be exceeded at a specified confidence level
- The Basel rules require commercial banks to hold capital to cover basically 3 times the average of the daily VAR at the 99% confidence level over a 10-day horizon (plus specific risk)
- Risk measures should be backtested using exception tests, counting the fraction of days losses exceeds VAR
- For well-diversified portfolios, risk goes down with the number of assets; for large and well-diversified portfolios, the average correlation becomes the main driver of risk; when the correlation is zero, portfolio risk goes down to zero.
- VAR tools include marginal VAR, which is the increase in VAR for a unit increase in the risk factor exposure, and component VAR, which is an additive decomposition of VAR; this is also marginal VAR times the size of the position
- Incremental VAR is the actual change in VAR if the position is dropped (which requires recomputing VAR)
- Volatility can be forecast well over a horizon of 1 day using GARCH models; GARCH models have mean reversion to a long run value
- The exponentially weighted moving average (EWMA) is a special case of GARCH with no mean reversion
- Correlations are more difficult to model due to dimensionality problems and the need to keep the covariance matrix positive definite; EWMA has a very simple structure for correlations when the decay is the same for all assets
- Factor models try to simplify the covariance matrix by reducing the number of independent dimensions; principal component analysis extracts these factors from the actual correlation matrix; portfolio risk can then be expressed in terms of exposures on the main factors and their risks; in well-diversified portfolios, residual risk goes to zero
- Joint distributions can be described by their marginal distribution as well as a copula; most widely used is the normal copula; this implies, however, weak dependencies in the tails

Market risk models:
- Systems require (1) position measurement, (2) modeling of risk factors, and (3) a risk engine that bring both together
- First step is pricing, or marking to market ($V_0$)
- Positions cannot be modeled individually; rather, they are mapped on risk factors
- Easiest approach includes local valuation using first derivatives $\partial V/\partial S$ and second derivatives $\partial^2 V/\partial S^2$
- Full valuation reprises all instruments, which is more precise but also slower
- Linear VAR is given by $\text{VAR}_1 = \Delta \text{VAR}(dS)$; quadratic VAR is $\text{VAR}_1 = 0.5 \Delta \text{VAR}(dS)^2$
- Long positions in options have positive gamma, and hence lower VAR than from the linear model
- VAR method 1: variance/covariance or delta/normal: Using linear mapping on the factors, compute portfolio variance $\Sigma x$, from which VAR is computed, using a normal distribution; main defects are lack of fat tails and non-linearities
- VAR method 2: historical simulation, where the vectors of historical changes in risk factors are applied to current value; portfolio is subject to full revaluation; main defect is short window
- VAR method 3: based on an analytical model of risk factors, run Monte Carlo simulations with full portfolio valuation; main defect is sampling variability and model risk
- Stress tests must be used as a complement to VAR, to consider events not in the VAR window or that have not happened yet; main issue is how to build relevant scenarios that lead to consistent joint movements in the risk factors; historical correlations can be used but assume stationary relationships

Credit risk models:
- Credit risk involves probability of default (or change in credit rating), loss given default, and exposure
- Probabilities of default can be estimated from default rates for different credit ratings
- Loss given default can be roughly estimated from traded prices of bonds right after default
- PD can also be estimated from market prices of bonds or stocks; these, however, lead to risk-neutral estimates
- Credit spreads include the RN PD times the LGD plus a premium for risk (liquidity, equity risk as proxied by beta)
- Structural, Merton-type models model stock prices as a call option on the value of the firm; distance to default depends on the market value of the firm, liabilities, and the volatility of firm values
- Exposure is the amount at risk, or claim on the counterparty upon default if positive
  o This is the positive value of a random variable whose distribution evolves over time
  o For bonds and loans, this is basically the notional amount
  o For int.rate swaps, initial exposure is zero, increases due to a dispersion effect, then decreases due to duration
  o For currency swaps, exposure increase until maturity due to the exchange of principals
- Exposure can be controlled by marking to market, or netting across contracts with the same counterparty
- The joint default process is usually modeled using latent variables for the asset values that have multivariate normal densities; credit migration or default is modeled using cutoff points for various transition probabilities; correlations are derived from equity correlations; tails are very sensitive to correlation and copula assumptions
The Basel II rules impose a credit risk charge either a (1) ratings-based, (2) foundation or (3) advanced internal ratings model. The charge roughly covers unexpected credit loss at a 99.9% confidence level over 1 year.

**Major Formulas**

### Derivatives:

- **Valuation of an outstanding forward contract:** 
  \[ V = S \exp(-y \tau) - K \exp(-r \tau) \]

- **Valuation of receive-fixed interest rate swap:** 
  \[ V = B(r; \text{notional}, \text{coupon}, \tau) - \text{FRN} \]

- **Valuation of receive-fixed foreign currency swap:** 
  \[ V = S($/\text{FC}) B^*(r*; \text{notional*}, \text{coupon*}, \tau) - B(r; \text{notional}, \text{coupon}, \tau) \]

### Market risk:

- **Cross-FX rate risk:** If \( \text{Cross-FX rate} \), then

- **Fixed-income risk using duration:**
  \[ \text{Duration} \]

- **Optimal number of futures contracts to sell for minimum-variance hedge:**

- **Optimal number of contracts for duration hedge:**

- **Option partial derivatives**
  - Delta is positive for calls, negative for puts; about 0.5 for ATM calls.
  - Gamma same for European calls and puts, highest for ATM short-term options.
  - Vega same for European calls and puts, highest for ATM long-term options.
  - \( |\Theta| \) is highest for ATM short-term options.

- **Using a linear approximation, a long position in an option is equivalent of delta times the underlying asset with debt.**

- **Long option positions have positive gamma and a shorter left tail, or less possibilities of large losses.**

- **GARCH model for conditional variance**
  \[ h_t = \alpha_0 + \alpha_1 (R_{t-1} - \mu)^2 + \beta_1 h_{t-1} \]

- **EWMA (exponentially weighted):**
  \[ h_t = (1 - \lambda) (R_{t-1} - \mu)^2 + \lambda h_{t-1} \]

- **The distribution of exceptions is binomial, with a normal approximation of**
  \[ z = (N-pT) / \sqrt{(p(1-p)T)} \rightarrow N(0,1) \]

### Portfolio tools:

- **Marginal risk**
  \[ \Delta \text{VAR} = (\text{VAR}/W) \beta_i = \alpha \sigma \beta_i = \alpha \sigma \rho_{i,p} \]

- **Component risk**
  \[ \text{CVAR}_i = \text{VAR}_p \times \beta_i \]

### Portfolio credit risk

- **Default-mode portfolio credit loss:**
  \[ CL = \sum_{i=1}^{N} b_i \times \text{EAD}_i \times \text{LGD}_i \]

- **Joint default event:**
  \[ E[b_A b_B] = \rho \sqrt{p_A(1-p_A)} \sqrt{p_B(1-p_B)} + p(A)p(B) \]

- **Cumulative default rate:**
  \[ C_N(R) = d_1 + (1-d_1)d_2 + \ldots + [\Pi_{i=1}^{N-1}(1-d_i)]d_N \]

- **Implied PD from credit spread:**
  \[ y^* - y \approx \pi (1 - f) \]

- **Merton structural model:**
  \[ \text{Stock} = \text{Call} \left( \text{Value of Firm}, \text{Strike=Debt Amount} \right) \]

- **Long corporate bond = long Treasury bond + short credit default swap.**

- **Volatility of firm value**
  \[ \sigma_S = N(d_1)(V / \sigma_V) \]

- **KMV distance to default:**
  \[ DD = \left[ \text{Value of firm assets} - (\text{ST debt + 0.5*LT debt}) \right] / \sigma_V \]

- **1-factor model for asset values:**
  \[ R_u = \sqrt{\rho M_1 + \sqrt{1-\rho} \varepsilon_u} \]

### Operational risk

- **“risk of loss resulting from inadequate or failed internal processes, people and systems, or from external events”**

- **Loss distribution approach combines:**
  - Loss frequency, or number of losses per year
  - Loss severity, or size of loss

### Regulatory requirements

- **Basel I Credit Risk Charge (CRC)**
  \[ \text{CRC} = \sum_i \text{K}_i \times \text{EAD}_i = 8\% \left( \sum_i \text{RW}_i \times \text{EAD}_i \right) = 8\% \text{(CRWA)} \]

- **Basel I and II capital ratio**
  \[ \text{Total Risk Capital} = \text{CRC} + \text{MRC} + \text{Op.Risk} \]

- **Basel exposure for derivatives**
  \[ CE = \text{NRV} + \text{AddOnFactor} \times N \times (0.4 + 0.6 \times NGR) \]

- **Basel Market Risk Charge (MRC) for trading book = General RC + Specific RC + Incremental RC**
  - **General RC = 3 times 10-day VAR at 99% confidence level**
  - **Basel III added a StressVAR and higher IRC, which increases the total MRC sharply**

- **Operational Risk Charge**
  - **Basic Indicator Approach**
    \[ K_{BIA} = EI \times \alpha \]
  - **Standardized Approach**
    \[ K_{SA} = \sum_i EI_i \times \beta_i \]
  - **AMA = UL = VAR(1 - year, 99.9\%) - EL**
- Advanced Measurement Approach
- Insurance can be used to cover up to 20% of capital